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# Foreword

Math Mammoth Grade 4 comprises a complete math curriculum for the fourth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

The main areas of study in Math Mammoth Grade 4 are:

1. Students develop understanding and fluency with multi-digit multiplication, and use efficient multiplication procedures to solve problems.
2. They develop understanding of division to find quotients involving multi-digit dividends (long division), and they solve word problems involving division, including division with a remainder.
3. Students develop an understanding of fraction equivalence and some operations with fractions. They learn to add and subtract fractions with same denominators, and to multiply a fraction by a whole number.
4. Students learn the concept of angle. They draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Additional topics we study are place value, time, measuring, graphs, and decimals.

This book, 4-B, covers division (chapter 5), geometry (chapter 6), fractions (chapter 7), and decimals (chapter 8). The rest of the topics are covered in the 4-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. Chapters 1, 2, and 3 should be studied in order, and Chapter 3 (multiplication) should be studied before Chapter 5 (division). However, you can be flexible with chapters 4 (time and measuring) and 6 (geometry), and schedule them earlier or later. Also, most lessons from chapters 7 and 8 (fractions and decimals) can be studied earlier; however the topic of finding parts with division should naturally be studied only after mastering division.
- Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your child study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for your child.
- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your child’s needs. You can use the skipped exercises later for review. For most children, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at <https://www.mathmammoth.com/faq-lightblue.php>

I heartily recommend that you view the full user guide for your grade level, available at <https://www.mathmammoth.com/userguides/>

And lastly, you can find free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>

*I wish you success in teaching math!*

*Maria Miller, the author*

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# Chapter 5: Division

## Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, average, divisibility, and problem solving. It is a long chapter, because division and long division are in focus in fourth grade. Therefore, feel free to mix the lessons from this chapter with lessons from some other chapter, essentially using the curriculum in a somewhat spiral manner. This is especially advisable if your student has difficulties retaining the material or starts feeling bored with these topics.

For further help in teaching these topics, check out the free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>. Remember not to automatically assign all the exercises but to adjust the amount according to the student's needs. The rest can be used later for review.

We start out by reviewing basic division facts by single-digit numbers (such as  $24 \div 4$  or  $56 \div 7$ ). After that, we study terminology of division and dividing numbers by whole tens and hundreds (such as  $400 \div 20$ ). Next students practice the order of operations again—this time with division as one of the operations.

Then we study the concept of remainder, preparing students for the upcoming lessons on long division. At first, the concept of remainder is presented visually. Soon, students solve simple division problems with a remainder, written with the long division symbol (or long division “corner”, as I like to call it).

Next comes a set of lessons intended to teach long division in several small steps. We start with divisions where each of the digits in the dividend (thousands, hundreds, tens, and ones) can be divided evenly by the divisor (for example,  $3096 \div 3$ ). As the next step, there is a remainder in the ones. Then, the divisions have a remainder in the tens. Finally, there is a remainder in the hundreds and in the thousands, and this completes the step-by-step learning process for long division. The lessons also include lots of word problems to solve.

After long division, we study the concept of average, which is a nice application of division, and problems that involve finding a fractional part of a quantity using division. For example, we can find  $\frac{3}{4}$  of a number by first finding  $\frac{1}{4}$  (dividing by 4) and then multiplying the result by 3. Students get help from visual bar models to solve the problems.

The last section deals with elementary number theory. We study basic divisibility rules (though not all of them), prime numbers, and finding all factors of a given two-digit number.

### The Lessons in Chapter 5

	page	span
Review of Division .....	10	3 pages
Division Terms and Division with Zero .....	13	2 pages
Dividing with Whole Tens and Hundreds .....	15	3 pages
Order of Operations and Division.....	18	2 pages
The Remainder, Part 1 .....	20	3 pages
The Remainder, Part 2 .....	23	2 pages
The Remainder, Part 3 .....	25	2 pages
Long Division 1 .....	27	4 pages
Long Division 2 .....	31	3 pages

Long Division 3 .....	34	4 pages
Long Division with 4-Digit Numbers .....	38	4 pages
More Long Division .....	42	3 pages
Remainder Problems .....	45	4 pages
Long Division with Money .....	49	2 pages
Long Division Crossword Puzzle .....	51	1 page
Average .....	52	3 pages
Finding Fractional Parts with Division .....	55	3 pages
Problems with Fractional Parts .....	58	2 pages
Problems to Solve .....	60	3 pages
Divisibility .....	63	4 pages
Prime Numbers .....	67	3 pages
Finding Factors .....	70	2 pages
Mixed Review Chapter 5 .....	72	2 pages
Review Chapter 5 .....	74	2 pages

## Helpful Resources on the Internet

You can also access this list of links at <https://l.mathmammoth.com/gr4ch5>

### DIVISION CONCEPT AND DIVISION FACTS

#### Fraction of...

Practice your ability to find a fraction of a given amount with this self-check exercise.

<https://www.transum.org/Maths/Exercise/Fractions/Default.asp?Level=2>

#### Patty's Paints Division

Help Patty paint cars by solving basic division questions. Lastly, drive your newly painted car in a fun race!

<https://www.multiplication.com/games/play/pattys-paints-division>

#### Bingo Game

Practice basic operations (addition, subtraction, multiplication, or division) with this fun online bingo game!

<https://www.mathmammoth.com/practice/bingo>

#### Flying High Race - Division

Race against other pilots while answering division problems. The faster you answer, the faster your plane goes!

<https://www.multiplication.com/games/play/flying-high-race-division>

#### Times or Divide Bingo

A useful class teaching resource on division and multiplication by 10 and 100. It includes decimals and is suitable for use on an interactive whiteboard.

<https://www.topmarks.co.uk/Flash.aspx?f=bingotimesordivide>

#### Leftovers—game with beads

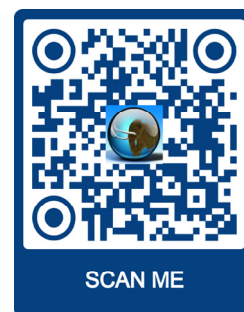
Practice division with this fun dice game!

<https://www.learn-with-math-games.com/long-division-games-for-the-classroom.html>

#### Area and Perimeter Builder

Build two-part rectangles with a given area. You're also given two fractions that indicate how to divide the area into parts. Choose "Game", "Level 5".

<https://www.mathmammoth.com/practice/area-builder>



### **Multiply & Divide Whole Numbers by 10, 100, 1,000 (Khan Academy)**

Practice multiplying and dividing by 10, 100, and 1,000 with this interactive online quiz.

<https://bit.ly/mult-div-10-100-1000>

### **Division with Remainders (Mental Math) — Online Practice**

Practice division with remainders with this ad-free online practice program at MathMammoth.com website. Also works as an offline program in most browsers. Includes the option for both timed and non-timed practice.

<https://www.mathmammoth.com/practice/division-remainder.php>

### **Order of Ops**

Save seven members of a Royal Family from prison by using your order of operation skills. The program uses a visual representation of a stairway to show how the mathematical expression gets shorter at each step.

<https://mrnussbaum.com/order-ops-online-game>

### **Free customizable worksheets for the order of operations**

Choose from five operations and parentheses. You can choose the number range, number of problems, and more.

[https://www.homeschoolmath.net/worksheets/order\\_of\\_operations.php](https://www.homeschoolmath.net/worksheets/order_of_operations.php)

### **ITP Remainders**

This ITP sets up an empty grid into which you can place counters. Removing or highlighting extra counters will change the calculation displayed.

[https://mathsframe.co.uk/en/resources/resource/67/itp\\_remainders](https://mathsframe.co.uk/en/resources/resource/67/itp_remainders)

### **Interpret Remainders**

Demonstrate your understanding of remainder in division with this interactive online quiz.

<https://www.khanacademy.org/math/cc-fourth-grade-math/division/4th-remainders/e/understanding-remainders?modal=1>

## **LONG DIVISION**

### **MathFrog Dividerama!**

Interactive long division practice. Guided help available.

<https://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml>

### **Mr. Martini's Classroom: Long Division**

An interactive long division tool.

<https://www.thegreatmartinicompany.com/longarithmetic/longdivision.html>

### **Drag-and-Drop Math**

Practice division interactively. Choose “Division”, 2-digit dividend, and 1-digit divisor.

<https://mrnussbaum.com/drag-n-drop-math-online>

### **Long Division Millionaire Game**

Learn to divide large numbers up to thousands. Can you answer all 15 questions?

<https://www.futuristicmath.com/games/6th-grade-long-division-millionaire-game.html>

### **Bike Racing Math Average**

Race your motorcycle against others while answering questions about average. Correct answers speed you up!

<https://www.mathnook.com/math/bike-racing-math-average.html>

### **Division Jump — board game**

Practice division of one-digit numbers into two, three, and four-digit numbers.

<https://www.learn-with-math-games.com/division-activities.html>

### **Long Division Quiz (Internet4Classrooms.com)**

Practice dividing four-digit numbers by single-digit numbers in this online quiz.

<https://bit.ly/long-division-quiz>

### **Double-Division.org**

Double-division is a form of the long division algorithm that takes away the guesswork of finding how many times the divisor goes into the number to be divided. Also called 1-2-4-8 division.

<http://www.doubledivision.org/>

## **Sample worksheet from**

<https://www.mathmammoth.com>

### **Short Division**

This is a web page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in your head and not written down.

<https://www.themathpage.com/ARITH/divide-whole-numbers.htm>

### **FACTORS AND PRIMES**

#### **Arrays and Factors**

Drag rectangles to show the factorizations of a given number on a grid.

<http://www.shodor.org/interactivate/activities/FactorizeTwo/>

#### **Find All the Factors of a Given Number — Online Practice**

An ad-free online practice program at MathMammoth.com. Also works as an offline program in most browsers.

<https://www.mathmammoth.com/practice/factorfind.php>

#### **Factor Game**

Choose a number from the game board, and your opponent gets all the numbers that are its proper factors. Adjust the number of rows and columns on the board to get a more challenging (and interesting) game. The game can be adapted to be played offline.

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Factor-Game/>

#### **Factor Pairs Puzzles**

Drag the numbers that are factors of the target number onto the gray squares to make the multiplications and inequalities correct. There are several puzzles to choose from.

[https://www.transum.org/Maths/Puzzles/Factor\\_Pairs/?Level=1](https://www.transum.org/Maths/Puzzles/Factor_Pairs/?Level=1)

#### **Pick the Primes**

Pick the prime numbered fruit from the tree by clicking on them as quickly as possible.

<https://www.transum.org/Maths/Game/Primes/Pick.asp>

#### **Prime Pairs Game**

A game for two players who take turns selecting two numbers that add up to a prime number.

[https://www.transum.org/Maths/Game/Prime\\_Pairs/](https://www.transum.org/Maths/Game/Prime_Pairs/)

#### **Factor Pair Up**

How well do you know your multiplication facts? Capture 3 or 4 products in a line before your opponent does.

[https://www.mathplayground.com/factor\\_pair\\_up.html](https://www.mathplayground.com/factor_pair_up.html)

#### **Flabbergasted Game**

Take turns choosing a number that is a factor or multiple of the number that the previous player chose. A game for two players, or for one player to play against the computer.

<https://www.transum.org/Maths/Game/Flabbergasted/>

#### **Product Game**

Choose factors, and the product of those gets colored in on the game board. The player who gets four products in a row wins. This game can easily be adapted to be played offline, with paper and colored pencils.

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Product-Game/>

#### **Prime Numbers and Composite Numbers, and Divisibility Rules—Mathsisfun.com**

Short lessons explaining divisibility tests, primes, and composite numbers. Each lesson includes interactive practice questions.

<https://www.mathsisfun.com/prime-composite-number.html>

<https://www.mathsisfun.com/divisibility-rules.html>

#### **Factoring Calculator**

This tool lists all the factors of a given number and shows an interesting visual that pairs the various factors of the number. You can even find all the factors of very large numbers, and it is fun to experiment with!

<https://www.dadsworksheets.com/factoring-calculator.html>

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## The Remainder, Part 3

**Example 1.** There were 238 children who participated in a special science class. They were divided into groups of 50. How many groups were there?

Instead of trying to divide  $238 \div 50$ , you can *add* to find the answer.

50 children make one group.	200 children make 4 groups.
100 children make 2 groups.	250 children make 5 groups.
150 children make 3 groups.	

This means they had four groups of 50 (a total of 200), and one other group of 38. In other words, the 238 children were grouped like this:  $50 + 50 + 50 + 50 + 38$ .

From this, we can write the division  $238 \div 50 = 4 \text{ R}38$ . Addition/multiplication will work just as well. In fact, multiplication is always used to solve division problems.

- A hundred school children traveled to a pool in buses. Each bus could hold 42 children. How many buses were needed?
- Jessica printed 73 pages of worksheets and put them into folders. Each folder could hold 20 pages.
 

How many folders did she need?

How many folders were full?
- A school has 77 first graders.
  - How many classes of 22 first graders could they make?
  - The school decided to put 20 first graders in each class. How many first-grade classes with 20 students will they have?
 

How many first graders are left over to form a class with less than 20?
- The gym leader divided 20 players into three teams, as evenly as possible. How many players were on each team?
- Divide and find the remainder by subtracting. Then, check your answers.

a.

$$6 \overline{)47}$$

b.

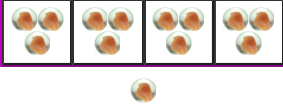
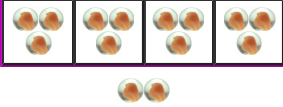
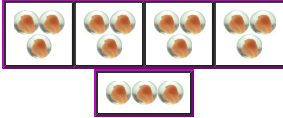
$$7 \overline{)58}$$

c.

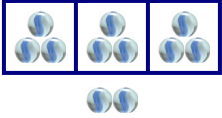
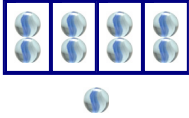
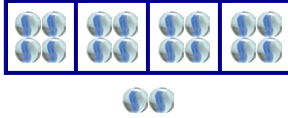
$$5 \overline{)44}$$

d.

$$9 \overline{)39}$$

		
$13 \div 3 = 4 \text{ R}1$	$14 \div 3 = 4 \text{ R}2$	$15 \div 3 = 5 \text{ R}0$
13 divided into groups of 3 makes 4 groups. One is left over.	Add one more marble. It is part of the leftovers.	Add one more. Now, instead of three “leftover” marbles, we can make one more group of 3!

6. First draw one more marble in each picture. Then check if you can make one more group or not. Then write a division sentence.

<b>a.</b> 	<b>b.</b> 	<b>c.</b> 
_____ $\div$ _____ = _____ R _____	_____ $\div$ _____ = _____ R _____	_____ $\div$ _____ = _____ R _____

7. Solve, and find a pattern.

<b>a.</b> $21 \div 5 =$ _____ R _____ $22 \div 5 =$ _____ R _____ $23 \div 5 =$ _____ R _____ $24 \div 5 =$ _____ R _____	<b>b.</b> $56 \div 8 =$ _____ R _____ $57 \div 8 =$ _____ R _____ $58 \div 8 =$ _____ R _____ $59 \div 8 =$ _____ R _____	<b>c.</b> $43 \div 7 =$ _____ R _____ $44 \div 7 =$ _____ R _____ $45 \div 7 =$ _____ R _____ $46 \div 7 =$ _____ R _____
--	--	--

8. Divide by 10. Indicate the remainder. Can you figure out a shortcut?

<b>a.</b> $29 \div 10 =$ _____ R _____ $30 \div 10 =$ _____ R _____ $31 \div 10 =$ _____ R _____	<b>b.</b> $78 \div 10 =$ _____ R _____ $79 \div 10 =$ _____ R _____ $80 \div 10 =$ _____ R _____	<b>c.</b> $54 \div 10 =$ _____ R _____ $55 \div 10 =$ _____ R _____ $56 \div 10 =$ _____ R _____
--	--	--

### Puzzle Corner

The number sentence that *checks* the division is given. Write the corresponding division sentence.

**a.**  $5 \times 3 + 1 = 16$

\_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_

**b.**  $7 \times 4 + 3 = 31$

\_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_

**c.**  $4 \times 30 + 3 = 123$

\_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_

# Long Division 1

## Divide hundreds, tens, and ones separately.

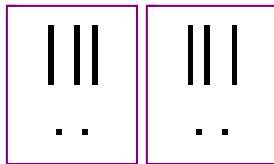
Write the dividend inside the long division “corner”, and the quotient on top.

$$\underline{64} \div 2 = ?$$

Divide tens and ones separately:

$$6 \text{ tens} \div 2 = 3 \text{ tens (t)}$$

$$4 \text{ ones} \div 2 = 2 \text{ ones (o)}$$



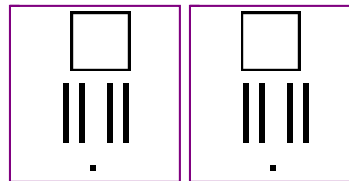
$$\begin{array}{r} \text{t o} \\ 3 \ 2 \\ \hline 2 \overline{) 6 \ 4} \end{array}$$

$$\underline{282} \div 2 = ?$$

$$2 \text{ hundreds} \div 2 = 1 \text{ hundred (h)}$$

$$8 \text{ tens} \div 2 = 4 \text{ tens (t)}$$

$$2 \div 2 = 1 \text{ (o)}$$



$$\begin{array}{r} \text{h t o} \\ 1 \ 4 \ 1 \\ \hline 2 \overline{) 2 \ 8 \ 2} \end{array}$$

1. Make groups. Divide. Write the dividend inside the “corner” if it is missing.

a. Make 2 groups



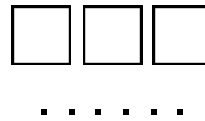
$$2 \overline{) 6 \ 2}$$

b. Make 3 groups



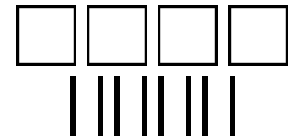
$$3 \overline{) \quad \quad}$$

c. Make 3 groups



$$3 \overline{) \quad \quad}$$

d. Make 4 groups



$$4 \overline{) \quad \quad}$$

2. Divide thousands, hundreds, tens, and ones separately.

a.  $4 \overline{) 8 \ 4}$

b.  $3 \overline{) 3 \ 9 \ 3}$

c.  $3 \overline{) 6 \ 6 \ 0}$

d.  $4 \overline{) 8 \ 0 \ 4 \ 0}$

e.  $3 \overline{) 6 \ 6}$

f.  $2 \overline{) 6 \ 0 \ 4 \ 2}$

g.  $3 \overline{) 3 \ 3 \ 0}$

h.  $4 \overline{) 4 \ 8 \ 0 \ 4}$

$\begin{array}{r} \text{h t o} \\ 0 \\ 4 \overline{) 248} \end{array}$ <p>Four does not go into 2. You can put zero in the quotient in the hundreds place or omit it. Four does go into 24, six times. Put 6 in the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 062 \\ 4 \overline{) 248} \end{array}$ <p>Five does not go into 3. You can put zero in the quotient. Five does go into 35, seven times.</p>
<p style="text-align: center;"><b><u>Explanation:</u></b></p> <p>The 2 of 248 is 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient will not have any whole hundreds.</p> <p>Then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result, 6 tens goes as part of the quotient.</p> <p>Check the final answer: <math>4 \times 62 = 248</math>.</p>	<p style="text-align: center;"><b><u>Explanation:</u></b></p> <p><math>3,000 \div 5</math> will not give any whole thousands to the quotient because the answer is less than 1,000.</p> <p>But 3 thousands and 5 hundreds make 35 hundreds together. You can divide <math>3,500 \div 5 = 700</math>, and place 7 as part of the quotient in the hundreds place.</p> <p>Check the final answer: <math>5 \times 701 = 3,505</math>.</p>
<p><b>If the divisor does not “go into” the first digit of the dividend, look at the <u>first two digits</u> of the dividend.</b></p>	

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. 
$$\begin{array}{r} 04 \\ 3 \overline{) 123} \end{array}$$

b. 
$$\begin{array}{r} \\ 4 \overline{) 284} \end{array}$$

c. 
$$\begin{array}{r} \\ 6 \overline{) 360} \end{array}$$

d. 
$$\begin{array}{r} \\ 8 \overline{) 248} \end{array}$$

e. 
$$\begin{array}{r} \\ 2 \overline{) 184} \end{array}$$

f. 
$$\begin{array}{r} \\ 7 \overline{) 427} \end{array}$$

g. 
$$\begin{array}{r} \\ 3 \overline{) 1833} \end{array}$$

h. 
$$\begin{array}{r} \\ 4 \overline{) 2404} \end{array}$$

i. 
$$\begin{array}{r} \\ 7 \overline{) 4970} \end{array}$$

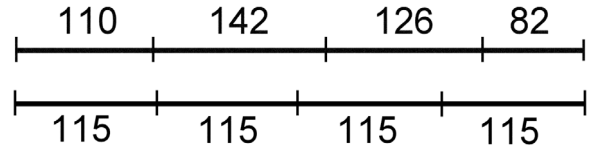
j. 
$$\begin{array}{r} \\ 5 \overline{) 4505} \end{array}$$

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# Average

The Millers went on a trip. The first day, they drove 110 miles, the second day, 142 miles, the third day, 126 miles, and the last day, 82 miles. The Millers drove a total of 460 miles.

In the diagram, we have put those distances as sticks one after another, though of course in reality they did not drive just straight stretches of roads.

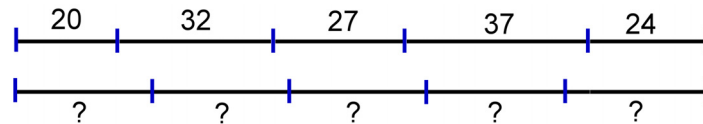


**IF** they had driven 115 miles each day, it would have totaled the same 460 miles.

*On average*, the Millers drove 115 miles a day, or their *average* daily mileage was 115 miles.

### What is the average of 20, 32, 27, 37, and 24?

First find the total by adding. Then, divide that into equal parts.



$20 + 32 + 27 + 37 + 24 = 140$ .  $140 \div 5 = 28$ . So, the **average** of 20, 32, 27, 37, and 24 is 28.

If these numbers were the ages of club members, we would say the average age of the members is 28 years. However, they could also be distances, weights, volumes, or just plain numbers.

- Judith's test scores were 78, 87, 69, and 86.  
Find her average score.
- John measured the temperature five times during a day.  
These are the results that he recorded:  
18°C, 22°C, 26°C, 23°C, and 16°C.  
Find the average temperature for the day.
- Dad drove 414 km in six hours.  
How many kilometers did he drive,  
on the average, in one hour?


**You can also figure the average backwards.**

*During a 20-hour drive from Denver to Dallas, Dad's average speed was 40 miles per hour. How far is Denver from Dallas?*

You can multiply to find the answer: 20 hours  $\times$  40 miles/hour = 800 miles.

(Note that in reality, he did not drive with a constant speed all of the time because he had to stop at crossings, slow down on curves, stop for a snack and so on. We do not know how much his speed varied on the trip. All we are given is that his *average* speed was 40 miles per hour. Of course the average speed was calculated by dividing the length of the trip by the total number of hours the trip took.)

4. The average weight of an egg is 55 grams. How much would a dozen eggs weigh?
5. For her hospital stay, Mom was charged an average of \$76 daily. What was the total cost of her one-week stay?
6. Mom's weekly grocery bills in June were \$234, \$178, \$250, and \$198. What was her average weekly grocery bill?
7. The children ran a race. These are the resulting times:

Ann	12 min
Judy	15 min
Rose	14 min
Elizabeth	19 min
Grace	12 min
Nancy	18 min

Michael	12 min
Greg	10 min
James	11 min
Caleb	15 min
Hans	17 min

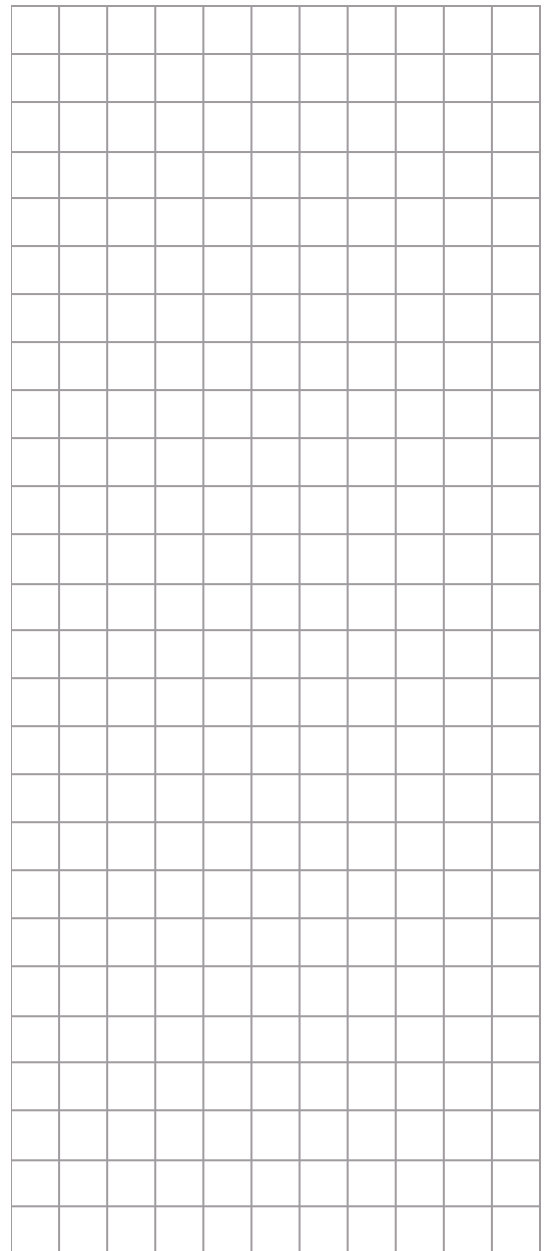
Find the girls' average running time and the boys' average running time *separately*.

Girls' average: \_\_\_\_\_

Boys' average: \_\_\_\_\_

Are boys or girls quicker on average?

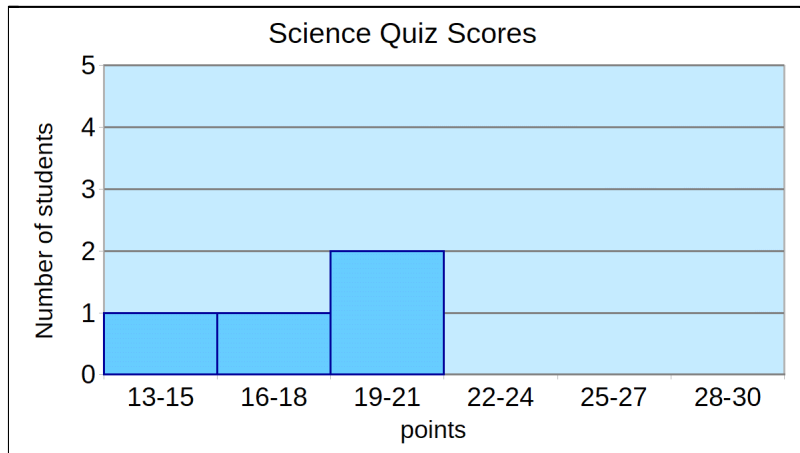
What is the difference of the two averages?



8. Here are the science quiz scores for ten fourth-graders: 24 20 24 16 28 30 14 22 23 19

a. Finish the frequency table and the graph. (“Frequency” refers to the number of students.)

Quiz score	Frequency
13-15	1
16-18	1
19-21	2
22-24	
25-27	
28-30	



b. Calculate the average score.

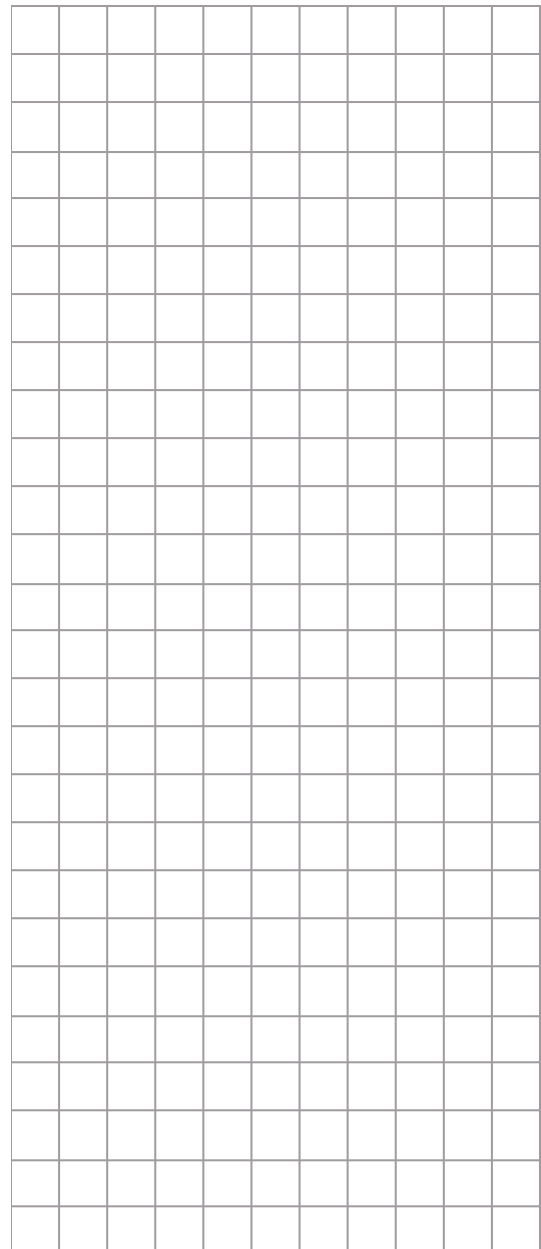
c. Both the graph and the average tell us what the “middle” or “typical” result in the test was. Explain how you can guess what the average is approximately, just using the graph.

9. These are the ages of the members of a bird watching club:

18 28 25 33 29 17 44 37 30

a. Calculate the average age.

b. The club gained a new member, 79-year-old Jim. What is the average age now?

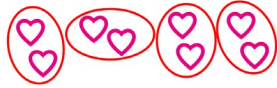


**Puzzle Corner**

If  $213 \div 17 = 12 \text{ R}9$ ,  
what is  $213 \div 12$ ?

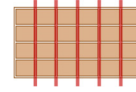


## Finding Fractional Parts with Division



These 8 hearts are divided into four equal groups.  
Each part is  $\frac{1}{4}$  (one-fourth) of the whole.

We can use division:  $8 \div 4 = 2$ . Each group has 2 hearts. So,  $\frac{1}{4}$  of 8 hearts is 2 hearts.

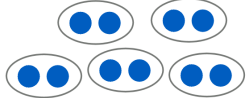
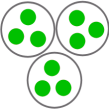
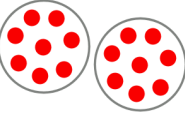

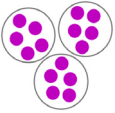



Mom divided 24 brownies into 6 equal parts. Each part is  $\frac{1}{6}$ th of the whole. How many pieces are in each part?





Divide to find out:  $24 \div 6 = 4$ . Four pieces. So,  $\frac{1}{6}$  of 24 brownies is 4 brownies.

**To find  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  etc. part of something, divide by 2, 3, 4, 5, etc. (respectively).**

1. Write a division sentence and a fractional part sentence.

<p>a. </p> <p>_____ <math>\div</math> 5 = _____</p> <p><math>\frac{1}{5}</math> of _____ is _____.</p>	<p>b. </p> <p>_____ <math>\div</math> _____ = _____</p> <p><math>\frac{1}{3}</math> of _____ is _____.</p>	<p>c. </p> <p>_____ <math>\div</math> _____ = _____</p> <p> of _____ is _____.</p>	<p>d. </p> <p>_____ <math>\div</math> _____ = _____</p> <p> of _____ is _____.</p>
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2. Write a fractional part sentence for each division sentence.

<p>a. <math>30 \div 5 = \underline{\quad}</math></p> <p> of 30 is _____.</p>	<p>b. <math>48 \div 6 = \underline{\quad}</math></p> <p> of _____ is _____.</p>	<p>c. <math>25 \div 5 = \underline{\quad}</math></p> <p> of _____ is _____.</p>	<p>d. <math>50 \div 5 = \underline{\quad}</math></p> <p> of _____ is _____.</p>
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3. Find a part. Also write a division sentence.

<p>a. <math>\frac{1}{6}</math> of 30 is _____.</p> <p><u>30</u> <math>\div</math> <u>6</u> = <u>5</u></p>	<p>b. <math>\frac{1}{7}</math> of 49 is _____.</p> <p>_____ <math>\div</math> _____ = _____</p>	<p>c. <math>\frac{1}{10}</math> of 250 is _____.</p> <p>_____ <math>\div</math> _____ = _____</p>
<p>d. <math>\frac{1}{2}</math> of 480 is _____.</p> <p>_____ <math>\div</math> _____ = _____</p>	<p>e. <math>\frac{1}{9}</math> of 1,800 is _____.</p> <p>_____ <math>\div</math> _____ = _____</p>	<p>f. <math>\frac{1}{5}</math> of 400 is _____.</p> <p>_____ <math>\div</math> _____ = _____</p>

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# Chapter 6: Geometry

## Introduction

We start our study of geometry by reviewing the third grade concepts of area and the perimeter of rectangles. Students also apply these concepts in various problems, including problems where they write simple equations and a problem where they explore all possible perimeters for a given area.

Note: Students will need a ruler and a protractor throughout the chapter.

The focus of the chapter is angles. Students learn about lines, rays, and angles; and about acute, right, obtuse, and straight angles. They learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations. The lesson *Estimating Angles* has an optional section on turning in an angle, which can be challenging, so feel free to omit it if you wish.

The lesson *Parallel and Perpendicular Lines* also ties in with the topic of angles, because two lines are perpendicular if they form a right angle. After that, we study parallelograms and other quadrilaterals in more detail, paying attention to their angles and lengths of sides.

We also study triangles and classify them according to their angles (acute, obtuse, or right triangles). Classifying triangles according to their sides (equilateral, isosceles, or scalene) will be studied in 5th grade. The last (and easy) topic in this chapter is line symmetry.

The lessons include quite a few drawing exercises which can be done on blank paper, in a notebook, or in the worktext (for most). Please stress to the student to always use a ruler and other proper tools, such as a protractor or a triangular ruler, so the drawings will be as accurate as possible. Some exercises may mention to only sketch something, in which case it is okay to not use any drawing tools.

Geometry is full of strange-sounding words. I suggest that student(s) keep a geometry notebook, where they draw picture(s) and text to explain every new concept or term. This will help them to remember those terms. They can also do the drawing exercises in the notebook. Encourage the students to be creative so that the notebook becomes their own special work. You can even give them credit for it.

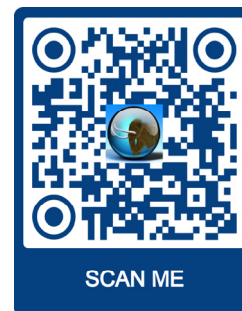
### The Lessons in Chapter 6

	page	span
Review: Area of Rectangles .....	81	3 pages
Problem Solving: Area of Rectangles .....	84	2 pages
Review: Area and Perimeter .....	86	4 pages
Lines, Rays, and Angles .....	90	3 pages
Measuring Angles .....	93	7 pages
Drawing Angles .....	100	2 pages
Estimating Angles .....	102	5 pages
Angle Problems .....	107	5 pages
Parallel and Perpendicular Lines .....	112	5 pages
Parallelograms .....	117	3 pages
Triangles .....	120	4 pages
Line Symmetry .....	124	3 pages
Mixed Review Chapter 6 .....	127	2 pages
Review Chapter 6 .....	129	4 pages

## Helpful Resources on the Internet

You can also access this list of links at <https://l.mathmammoth.com/gr4ch6>

*DISCLAIMER: We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*



### AREA AND PERIMETER

#### Free Worksheets for Area and Perimeter

Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property.

[https://www.homeschoolmath.net/worksheets/area\\_perimeter\\_rectangles.php](https://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php)

#### Area & Perimeter of Rectangles Word Problems

Practice finding the area and perimeter of rectangles with these interactive word problems.

<https://cutt.ly/area-perimeter-of-rectangles-word-problems>

#### Shape Explorer

Find the perimeter and area of odd shapes on a rectangular grid.

<http://www.shodor.org/interactivate/activities/ShapeExplorer/>

#### Area and Perimeter of a Rectangle

Test your problem solving abilities with these questions about area and perimeter of rectangles.

[https://www.transum.org/Software/SW/Starter\\_of\\_the\\_day/students/Oblongs.asp?Level=1](https://www.transum.org/Software/SW/Starter_of_the_day/students/Oblongs.asp?Level=1)

#### Area and Perimeter Builder

Create your own rectangular shapes using colorful blocks and explore the relationship between perimeter and area. You can choose to show the side lengths to understand how a perimeter works. You can also use two work areas (grids) to compare the area and perimeter of two shapes side-by-side. Lastly, challenge yourself in the game screen to build shapes or find the area of various figures.

<https://www.mathmammoth.com/practice/area-builder>

#### Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz, about the area and perimeter of rectangles, triangles, and trapezoids. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when the perimeter/area is given.

<https://www.thatquiz.org/tq-4/?-j1200b-lc-p0>

#### Area: Missing Side Length Quiz

Practice finding the value of the unknown side in this 10-question quiz.

<https://www.thatquiz.org/tq-4/?-j8001-lc-p0>

#### Zoo Designer

Use your knowledge of how to calculate area and perimeter to design the enclosures for animals at the zoo.

<https://mrnuessbaum.com/zoo-designer-online-game>

### ANGLES

#### Angles

A page about angles that includes an illustrative tool and interactive practice questions.

<https://www.mathsisfun.com/angles.html>

#### Angles and Their Measures Matching Game

Practice matching angles to their angle measures in this interactive online game.

<https://www.mathmammoth.com/practice/angles-matching>

Sample worksheet from  
<https://www.mathmammoth.com>

### **Using a Protractor**

Investigate angles and the use of protractors.

<https://www.mathsisfun.com/geometry/protractor-using.html>

### **Measuring Angles**

Practice measuring angles with a protractor.

<https://www.mathplayground.com/measuringangles.html>

### **Draw Angles - Khan Academy**

Use a protractor to construct angles.

<https://www.khanacademy.org/math/on-sixth-grade-math/on-geometry-spatial-sense/on-angles-polygons/e/drawing-angles>

### **Decompose Angles**

Test your knowledge of angles with this interactive online quiz.

<https://www.khanacademy.org/math/cc-fourth-grade-math/imp-geometry-2/imp-decomposing-angles/e/decomposing-angles?modal=1>

### **Playground**

Squirt water on a given target by setting the correct angle.

<https://www.free-training-tutorial.com/angles/playground/playground.html>

### **Estimating Angles**

Estimate the target angle. The closer you get to the target angle the more points you will score.

<https://nrich.maths.org/1235>

### **Draw Perpendicular and Parallel Lines - Interactive**

Learn about perpendicular and parallel lines and practice drawing them.

<https://www.mathsisfun.com/perpendicular-parallel.html>

### **Identify Parallel and Perpendicular Lines**

Classify the lines as parallel, perpendicular, or neither.

<https://cutt.ly/identify-parallel-perpendicular>

### **Angle Drag**

Drag the line to make a given angle, then measure using a protractor.

<https://mathsframe.co.uk/en/resources/resource/591/Angle-Drag>

### **Sectors**

Practice fitting sectors together to make complete circles in this interactive activity.

[https://www.transum.org/Software/SW/Starter\\_of\\_the\\_day/starter\\_February13.ASP](https://www.transum.org/Software/SW/Starter_of_the_day/starter_February13.ASP)

### **Measuring Angles**

Measure the size of the given angles to within two degrees of their actual value.

[https://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Measuring\\_Angles.asp](https://www.transum.org/software/SW/Starter_of_the_day/Students/Measuring_Angles.asp)

## **SHAPES / POLYGONS**

### **Polygon People**

Name the polygons and other geometrical shapes that make up the Polygon People.

[https://www.transum.org/Maths/Display/Polygon\\_People/Default.asp?Level=1](https://www.transum.org/Maths/Display/Polygon_People/Default.asp?Level=1)

### **Polygon Vocabulary**

A matching game.

<https://www.quia.com/cc/2758.html>

### **Interactive Quadrilaterals**

See all the different kinds of quadrilaterals “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.

<https://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

## **Sample worksheet from**

<https://www.mathmammoth.com>

### **Interactive Parallelogram**

Drag the parallelogram and learn about its properties, angles, and sides.

<https://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php>

### **Interactive Triangles Tool**

Read about triangles, and then play with them to become familiar with them from all angles.

<https://www.mathsisfun.com/geometry/triangles-interactive.html>

### **Triangles Splat**

“Shoot” the triangles as their names appear on the screen. Choose “Right, Acute, Obtuse”.

<https://www.sheppardsoftware.com/math/geometry/triangle-splat-game/>

### **Classify Triangles by Angles**

Practice classifying triangles with this interactive multiple-choice quiz.

<https://www.khanacademy.org/math/cc-fourth-grade-math/plane-figures/imp-classifying-triangles/e/identifying-triangles-by-angles>

### **Identify Line Symmetry**

Test your knowledge of line symmetry with this interactive online quiz.

[https://www.khanacademy.org/math/cc-fourth-grade-math/plane-figures/imp-line-of-symmetry/e/axis\\_of\\_symmetry?modal=1](https://www.khanacademy.org/math/cc-fourth-grade-math/plane-figures/imp-line-of-symmetry/e/axis_of_symmetry?modal=1)

### **Symmetry Shapes Shoot**

Practice identifying symmetrical shapes by clicking on them.

<https://www.sheppardsoftware.com/math/geometry/symmetry-game/>

### **Line Shoot**

Learn about lines of symmetry the fun way in this line-shoot geometry math game.

<https://www.sheppardsoftware.com/math/geometry/symmetry-line-game/>

## **GENERAL**

### **Interactivate! Tessellate**

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

### **Patch Tool**

An online activity where the student designs a pattern using geometric shapes.

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Patch-Tool/>

### **Pattern Blocks**

Explore shapes and patterns with this online math manipulative.

<https://www.coolmath4kids.com/manipulatives/pattern-blocks>


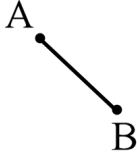
### **Geometry Worksheets**

Worksheets about complementary and supplementary angles, parallel, perpendicular, and intersecting lines, types of angles, basic shapes, area and perimeter of rectangles, and parts of a circle.

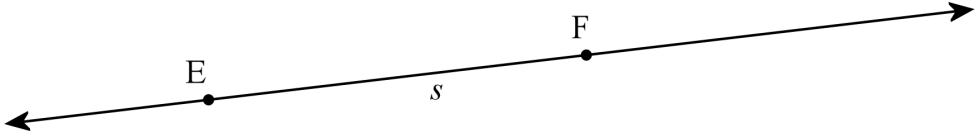
<https://www.dadsworksheets.com/worksheets/basic-geometry.html>

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# Lines, Rays, and Angles

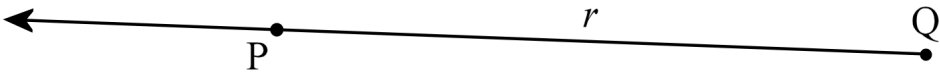
<p>This is point A. (Points are named with capital letters.)</p>	 <p>A</p>	<p>This is a <b>line segment</b>. We write this as line segment AB or line segment <math>\overline{AB}</math>.</p>	
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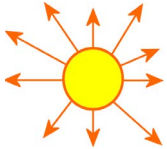
**A line** has no beginning point or end point. Imagine it continuing indefinitely in both directions. We can illustrate that by little arrows on both ends.



A line is named using two points on it. This is line EF or line  $\overleftrightarrow{EF}$  (note the arrows on both ends). Or, we can name a line using a lowercase letter: this is line *s*.


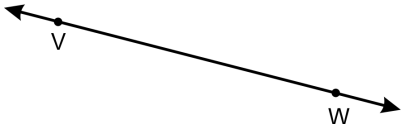
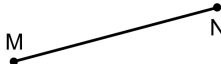
**A ray** starts out at a point but continues on indefinitely, without ending. We can show that by drawing an arrow at one end of the ray. Think of the sun's rays!





A ray is named using its starting point and one other point on the ray: this is ray QP or ray  $\overrightarrow{QP}$  (note the one arrow). Or, we can name a ray using a lowercase letter: this is ray *r*.

1. Write the name if each figure is a line, ray, or a line segment.

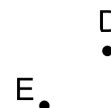
 <p>a. _____</p>	 <p>b. _____</p>	 <p>c. _____</p>
---	--	---

2. a. Draw the ray BD.



b. Draw the line AB.

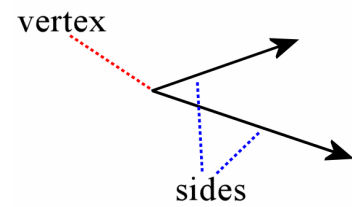
c. Draw the line segment ED.



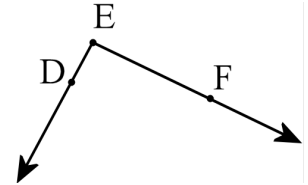


What is an angle? Many people think that an angle is some kind of slanted line. However, in geometry, **an angle** is made up of **TWO RAYS that have the same beginning point**.

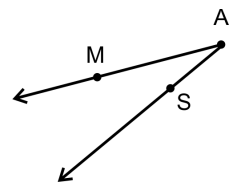
That point is called the **vertex**, and the two rays are called the **sides** of the angle.



To name an angle, we use three points, listing the vertex in the middle. This is angle DEF or  $\angle DEF$ . We can use the symbol  $\angle$  for angle.

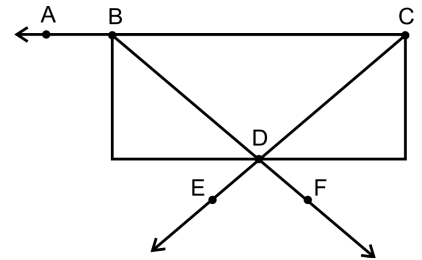


3. Name the angle.



4. **a.** Find the angle formed by the rays DE and DF.  
How do we name it?

**b.** Find the angle formed by the rays CA and CE.  
How do we name it?



5. **a.** Draw two points, D and E. Then draw line DE.

**b.** Draw point *Q* *not* on the line.

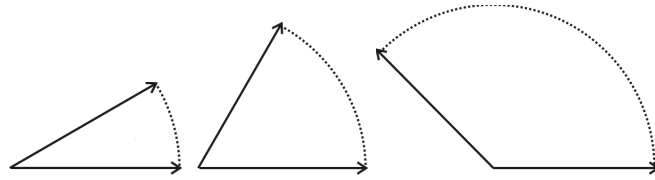
**c.** Draw rays DQ and EQ.

**d.** Find angles EDQ and DEQ in your drawing.

### Angles “open up”

Take two pencils to illustrate the two rays which make the two sides of an angle. Set them side by side to show a zero angle!  $\implies$

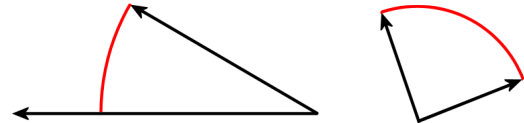
Then start “opening up” the angle. Keep one pencil stationary while you rotate the other. As the angle opens up, an imaginary arc of a circle is drawn.



### Telling the size of angles

Which of these two angles is bigger?

Do not look at how LONG the sides of the angle are. Remember, the sides of an angle are rays, and rays go on indefinitely (even if they are drawn short).

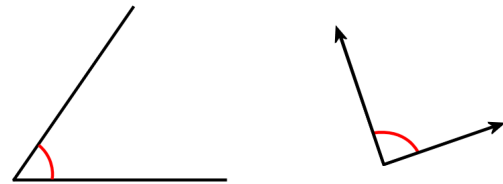


Instead, look at how much each angle has opened, or how big a part of a circle the sides have drawn. The second angle (on the right) is bigger.

The **size of an angle** is determined by **how much it has opened as compared to the whole circle**.

Many times the arrows are omitted from the rays, and the arc of the circle is drawn as a tiny arc near the vertex. Even that is not necessary.

Which of these is the bigger angle?  
Again, the second one is bigger.



6. Which angle is bigger? You can use pencils to help!

<p>a.</p> <p>OR</p>	<p>b.</p> <p>OR</p>	<p>c.</p> <p>OR</p>
<p>d.</p> <p>OR</p>	<p>e.</p> <p>OR</p>	<p>f.</p> <p>OR</p>

#### New Terms & Symbols

- *line segment*
- *ray*
- *line*
- *angle*

Note: In the next lesson, students will need a *protractor* to measure angles.

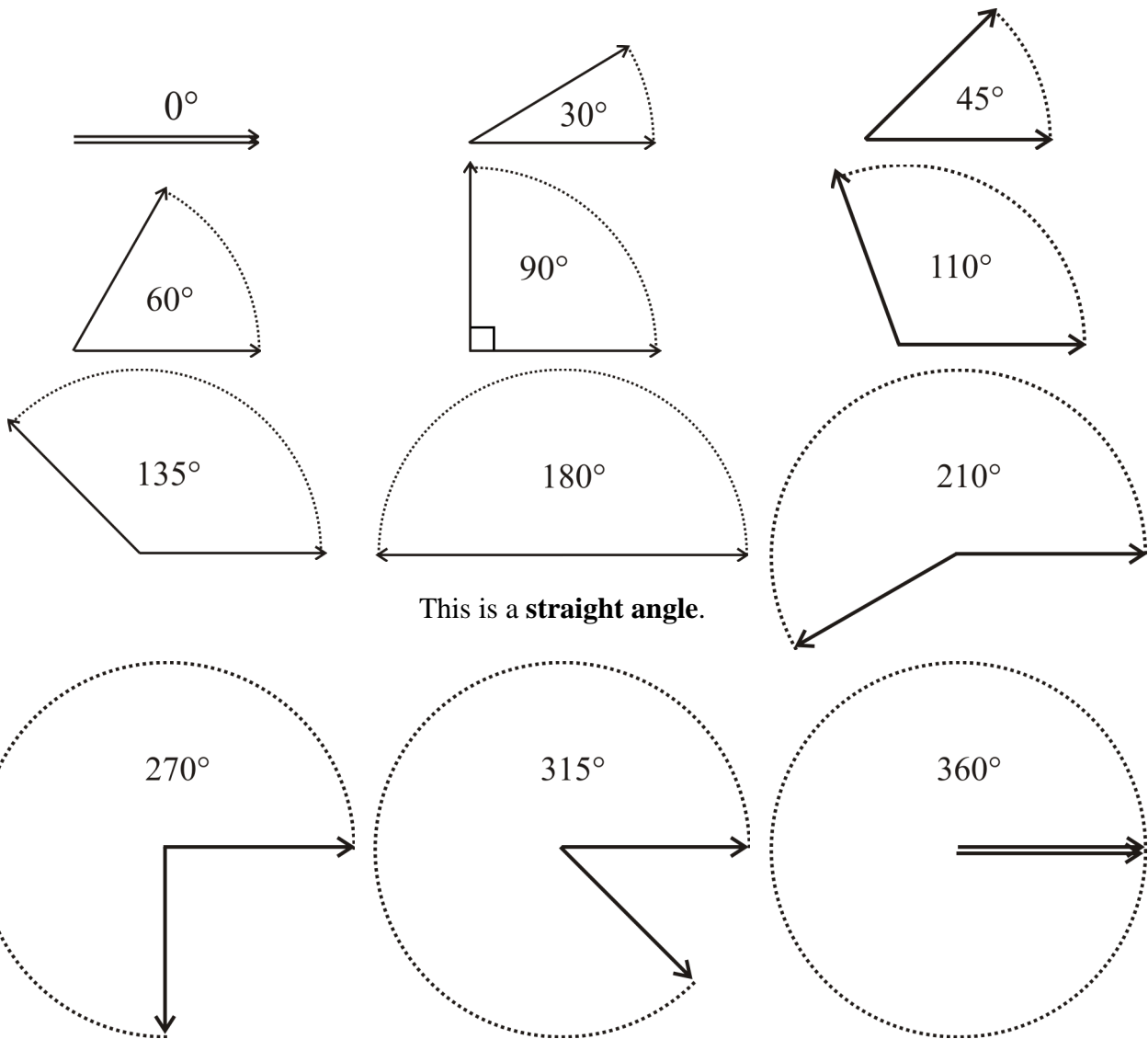
# Measuring Angles

Remember how one side of the angle traces out a circular arc°. We use that circle to measure how big the angle is. We look at how much the angle has “opened” as compared to the full circle.

Angles are measured in **degrees**. The symbol for degrees is a small raised circle: °.

- **The full circle is 360° (360 degrees).**
- A half circle (a straight angle) is 180°.
- A quarter of a circle (a right angle) is 90°.

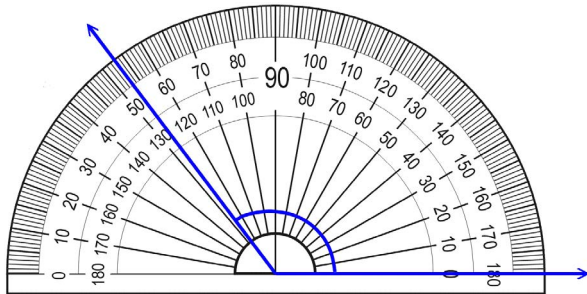
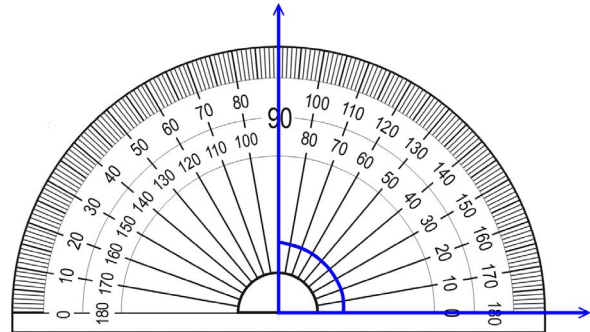
Show the angles below using two pencils. Try to “see” the circle that is traced in the air.



This is a 1-degree angle — it is 1/360 part of the full circle!

### How to use a protractor to measure angles

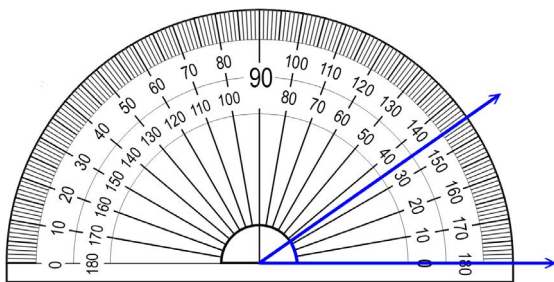
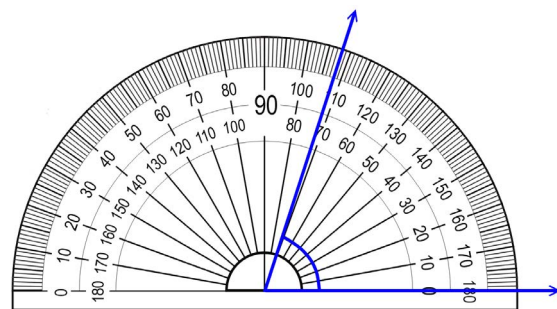
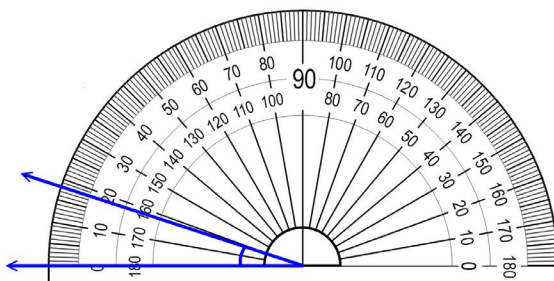
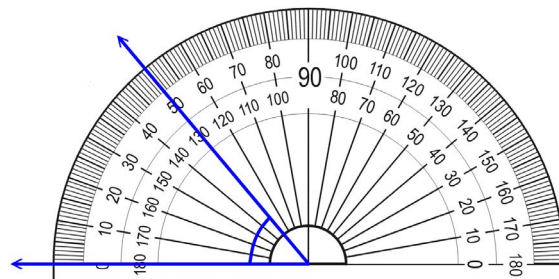
1. Place the midpoint of the protractor on the vertex of the angle.
2. Line up one side of the angle with the zero line of the protractor (where you see the number 0).
3. Read the degrees where the other side crosses the number scale.

an obtuse angle;  $127^\circ$ a right angle;  $90^\circ$ 

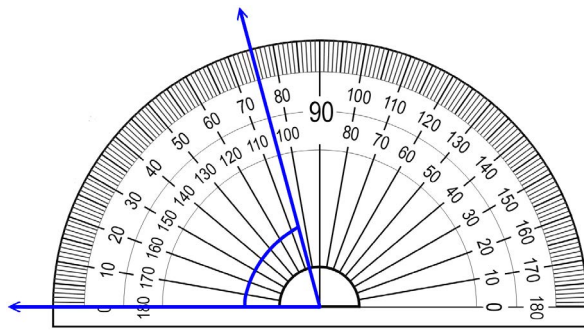
Make sure you read from the right set of numbers. A protractor has two sets of numbers: one set goes from 0 to 180, and the other set from 180 to 0. Which one you read depends on how you place the protractor in relation to the angle. One of the sides of the angle is lined up with one of the zero lines of the protractor. You read the set of numbers that starts with *that* zero.

In the examples above we lined up one of the sides of the angle with the zero of the lower set of numbers, so we need to read the lower set of numbers.

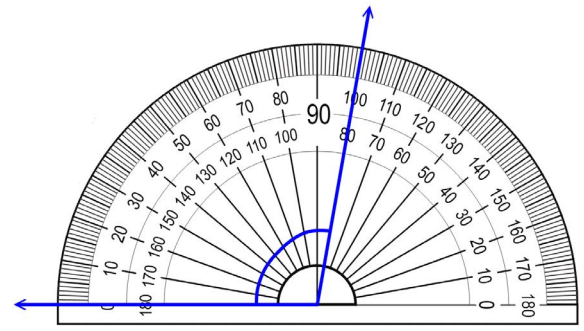
1. Measure the angles.

a. \_\_\_\_\_ $^\circ$ b. \_\_\_\_\_ $^\circ$ c. \_\_\_\_\_ $^\circ$ d. \_\_\_\_\_ $^\circ$

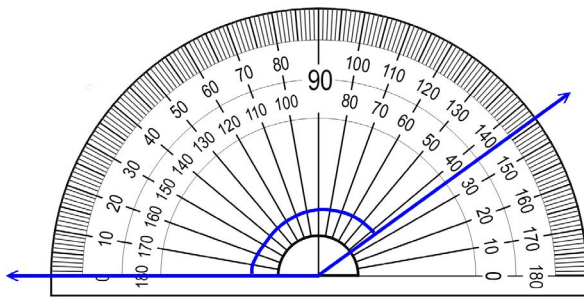
2. Measure the angles.



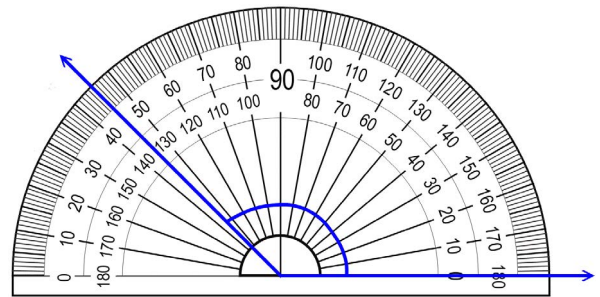
a. \_\_\_\_\_ °



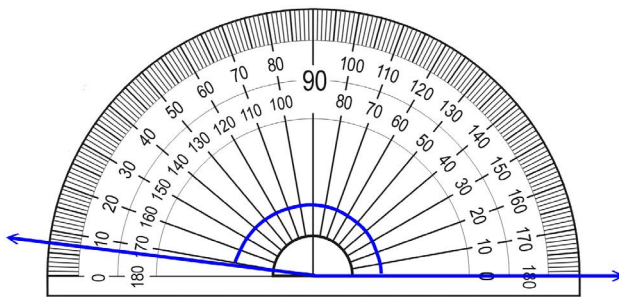
b. \_\_\_\_\_ °



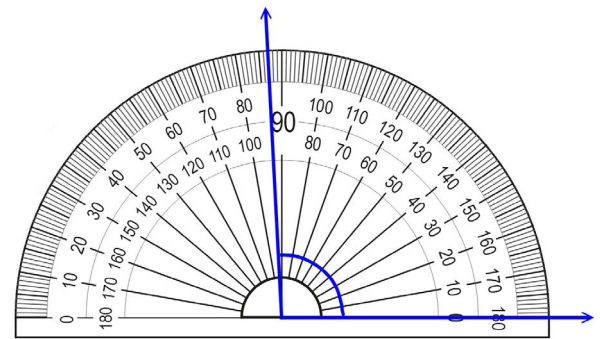
c. \_\_\_\_\_ °



d. \_\_\_\_\_ °



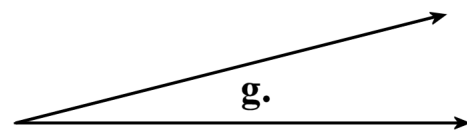
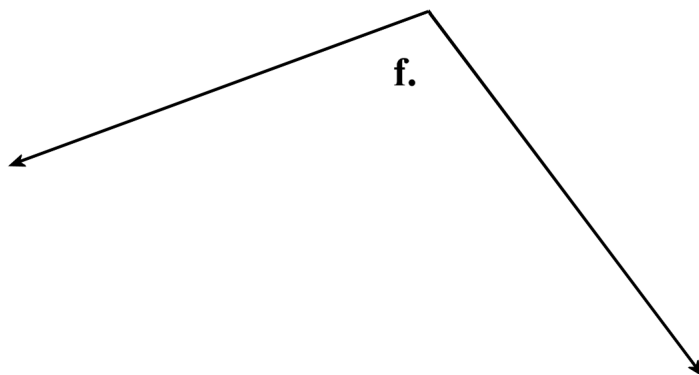
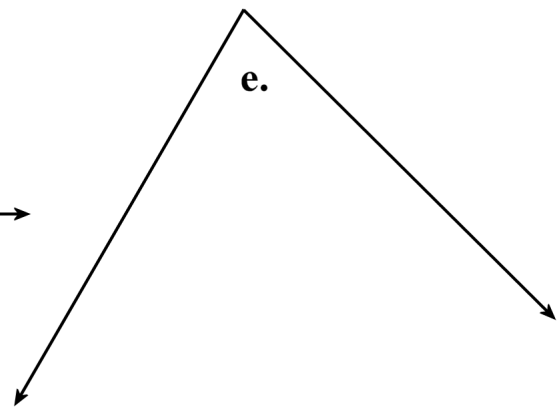
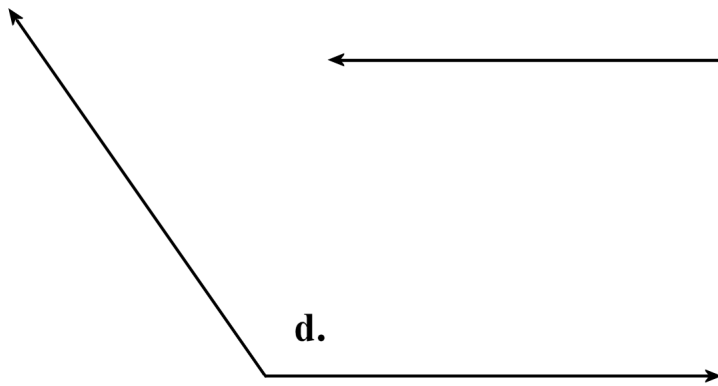
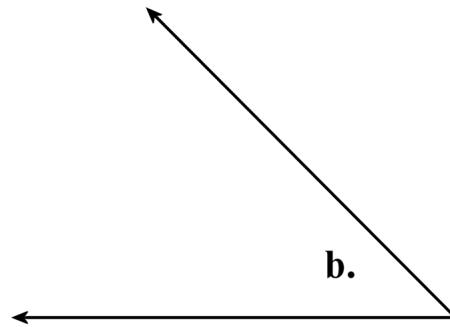
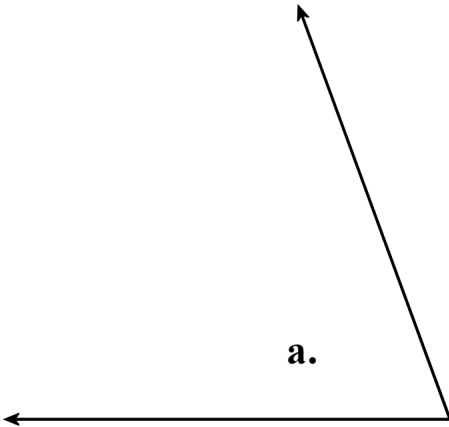
e. \_\_\_\_\_ °

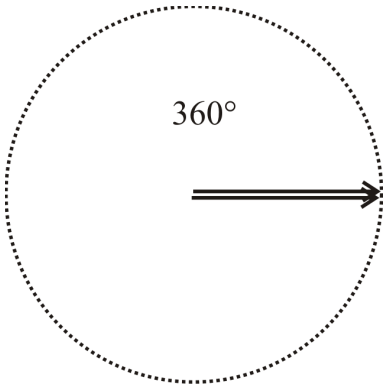
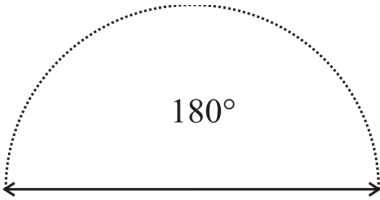
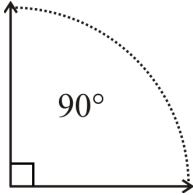
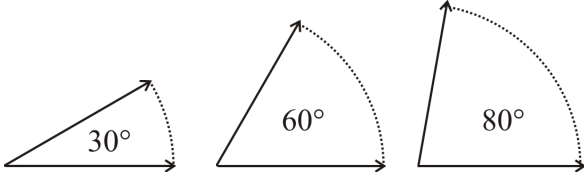
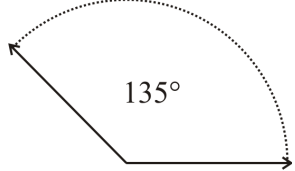


f. \_\_\_\_\_ °

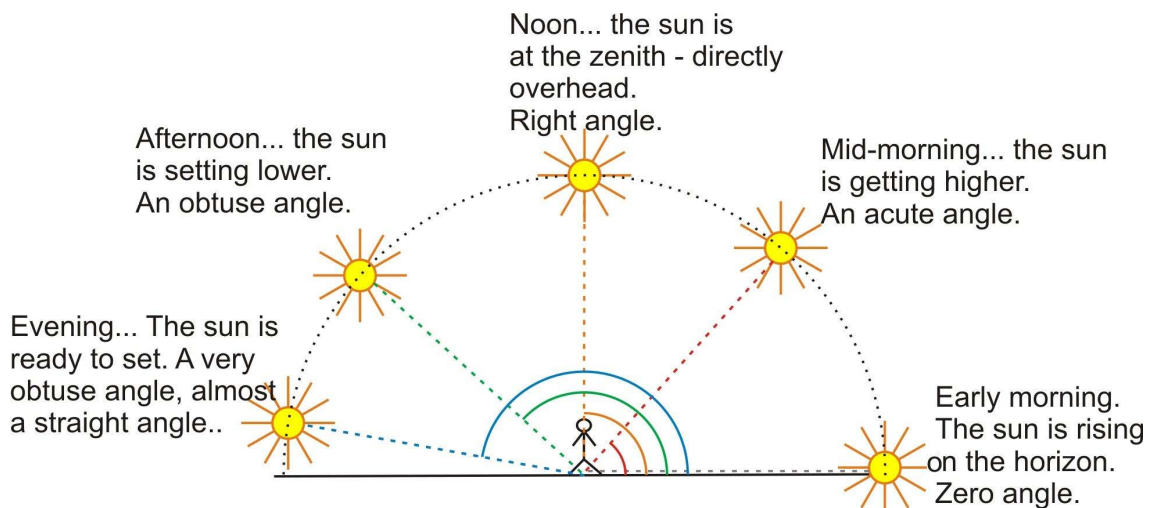
3. Tosha measured an acute angle and got  $146^\circ$ . The teacher pointed out that she had read the wrong set of numbers on the protractor. What is the correct measure for her angle?

4. Measure the following angles using your own protractor. If you need to, make the sides of the angles longer with a ruler.



Angle terminology		
 <p style="text-align: center;"><math>360^\circ</math></p>	 <p style="text-align: center;"><math>180^\circ</math></p>	 <p style="text-align: center;"><math>90^\circ</math></p>
<p>Here the pencil has drawn a full circle. This angle measures 360 degrees (<math>360^\circ</math>).</p>	<p>This angle is half of the full circle, so it measures <math>180^\circ</math>. It is called <b>the straight angle</b>.</p> <p>Your two pencils (rays) are lying down flat or <i>straight</i> on the floor.</p>	<p>This is one-fourth of the full circle, so it is <math>90^\circ</math>. It is called <b>the right angle</b>. Table and book corners are right angles.</p> <p>We often mark a right angle with a little corner.</p>
 <p style="text-align: center;"><math>30^\circ</math>      <math>60^\circ</math>      <math>80^\circ</math></p>		 <p style="text-align: center;"><math>135^\circ</math></p>
<p>In each of these pictures the angle is opened more and more and keeps getting bigger. The arc of the circle is larger.</p> <p>These angles are <b>acute angles</b>, which means they are less than a right angle (less than <math>90^\circ</math>). Think of acute angles as <i>sharp</i> angles. If someone stabbed you with the vertex of an acute angle, it would feel sharp.</p>		<p>This is an <b>obtuse angle</b>: it is more than a right angle, yet less than a straight angle.</p> <p>Think of obtuse angles as <i>dull</i> angles.</p>

Here is another way of thinking about angles. Think of a SUN rising in the morning on the horizon, gradually getting higher, and traveling through the sky along an arc of a circle.



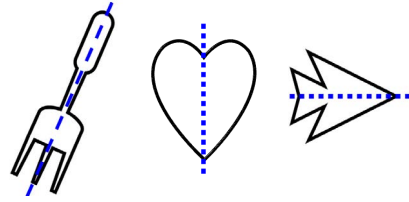
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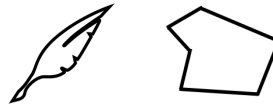
# Line Symmetry

These figures are **symmetrical** in relation to the dashed line.  
The line is called a **symmetry line**.

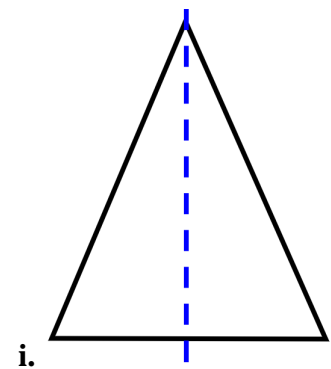
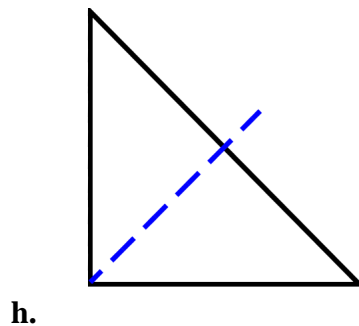
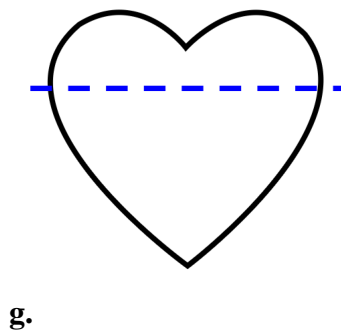
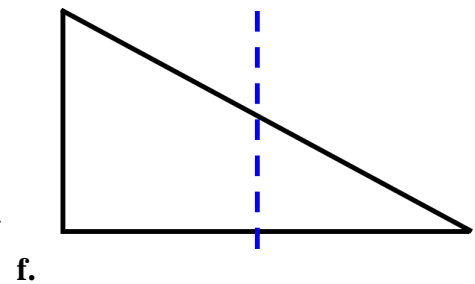
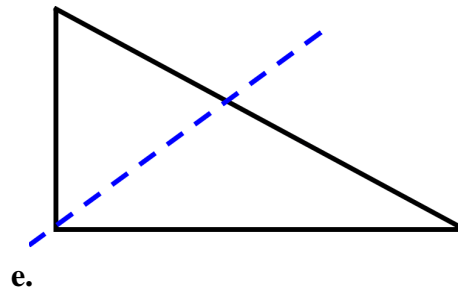
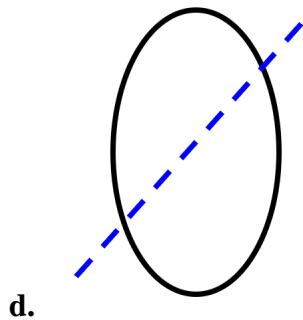
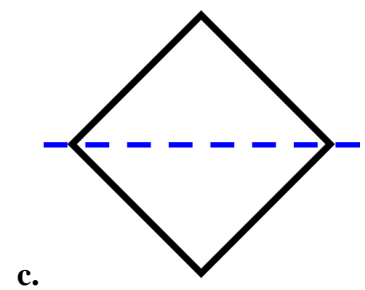
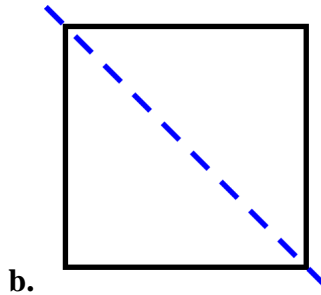
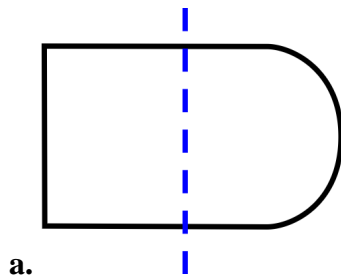
Imagine that you FOLDED the figure along the symmetry line. Then both sides would exactly meet. Or, if you placed a mirror along the symmetry line, you would see the other half of the figure reflected in the mirror.



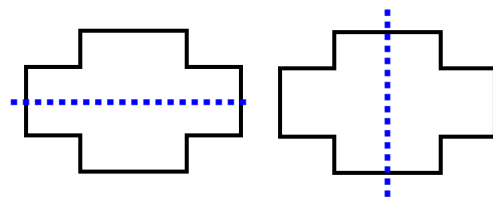
Many figures are not symmetrical at all.  
You cannot draw a symmetry line in them.



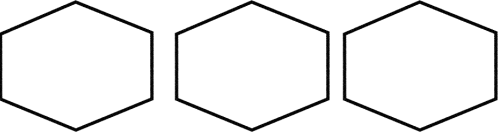
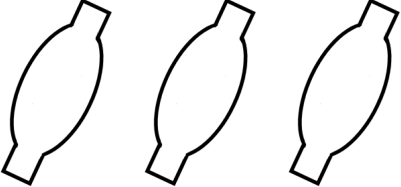
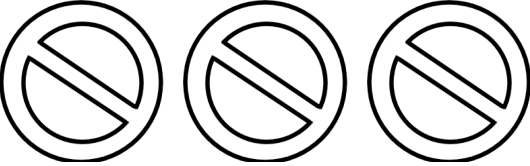
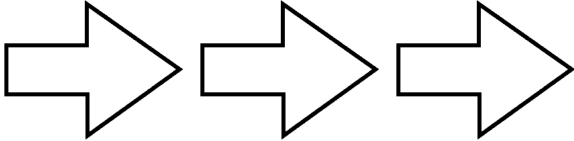
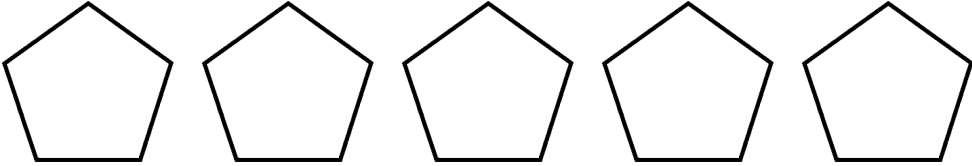
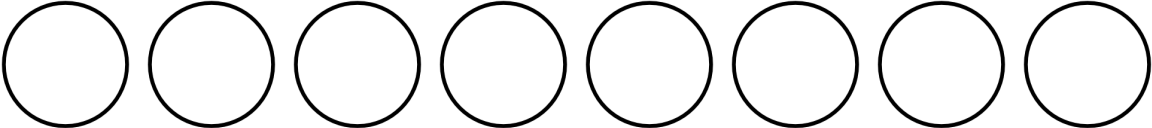
1. Is the line drawn a symmetry line for the figure? You can cut out the images and fold them along the dashed line to check.



Some shapes can be folded in two different ways so that the sides meet. The cross-shape on the right has *two* different symmetry lines.

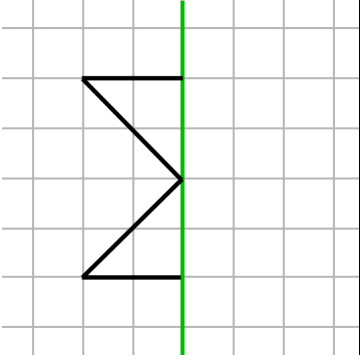
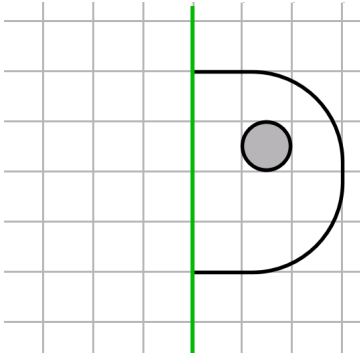
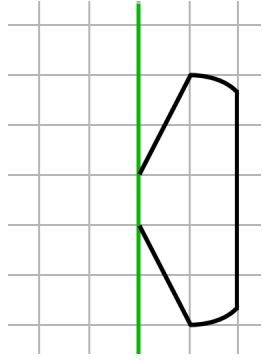

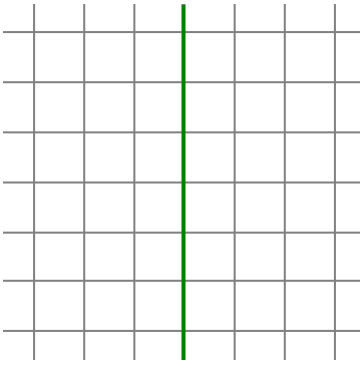



2. Draw as many different symmetry lines as you can in these shapes.

<p>a.</p> 	<p>b.</p> 
<p>c.</p> 	<p>d.</p> 
<p>e.</p> 	
<p>f.</p> 	

3. Write the capital letters through which you can draw a symmetry line. Draw the symmetry lines in them.

4. Draw a mirror image in the symmetry line to get a symmetrical figure.

<p>a.</p> 	<p>b.</p> 	<p>c.</p> 
<p>d.</p>  <p>Continue the pattern. Then draw its mirror image.</p>	<p>e.</p>  <p>Draw your own design and find its mirror image.</p>	<p>f.</p>  <p>Draw your own design and find its mirror image.</p>

5. Look for logos on food products, cars, stores, magazines, and so on. Find at least three logos that have symmetry. Sketch them below. Answer the questions for each logo.

- Does the logo employ a square, a rectangle, a triangle, a circle, or some other basic geometric figure in some way?
- Does it have any symmetry?

#### New Terms & Symbols

- *symmetrical*
- *symmetry line*

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# Chapter 7: Fractions

## Introduction

In third grade, students have studied equivalent fractions and compared some easy fractions. In fourth grade, it is time to expand their knowledge of fraction topics. We study:

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (sums where the denominators are the same, such as  $\frac{5}{6} + \frac{3}{6}$  or  $1\frac{2}{3} + 2\frac{1}{3}$ )
- equivalent fractions (for example,  $\frac{2}{3} = \frac{8}{12}$ )
- comparing fractions
- multiplying a fraction by a whole number (for example  $5 \times \frac{1}{2}$ )

Then in fifth grade, students tackle *all* four operations with fractions. This chapter is laying groundwork for that. The lessons here are important also because they are the basis for understanding decimal numbers, which is the topic of the next chapter.

In this grade, we continue studying fractions and their operations with the help of visual models. In addition to the visuals in the lessons, you can optionally also use fraction manipulatives, but they are not required.

Visual models help children build a strong conceptual understanding of fraction operations. While we do study some actual rules of fraction arithmetic in this chapter, we also want to avoid presenting fraction math as a list of computational rules to be learned by rote memory. If students only memorize these rules, then they will also easily confuse them (eventually), because there are so many of them. The rules become *shortcuts* for ideas that are already understood, but we don't want to start with them. The goal is to let the ideas and concepts "sink in" first, and then study the shortcuts.

A friendly reminder: don't automatically assign all the exercises. As always, use your judgment.

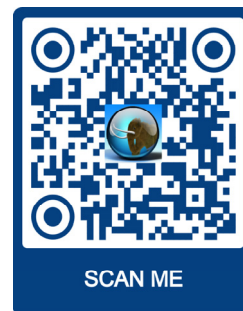
### The Lessons in Chapter 7

	page	span
One Whole and Its Fractional Parts .....	137	3 pages
Mixed Numbers .....	140	4 pages
Mixed Numbers and Fractions .....	144	3 pages
Adding Fractions .....	147	2 pages
Adding Mixed Numbers .....	149	3 pages
Equivalent Fractions .....	152	5 pages
Subtracting Fractions and Mixed Numbers ....	157	4 pages
Comparing Fractions .....	161	4 pages
Multiplying Fractions by Whole Numbers .....	165	3 pages
Practicing With Fractions .....	168	2 pages
Mixed Review Chapter 7 .....	170	2 pages
Review Chapter 7 .....	172	2 pages

## Helpful Resources on the Internet

You can also access this list of links at <https://l.mathmammoth.com/gr4ch7>

*DISCLAIMER: We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*



### FRACTIONS AND MIXED NUMBERS

#### Build a Fraction

Build fractions from shapes and numbers to earn stars in this fractions game or explore in the Fractions Lab.  
<https://phet.colorado.edu/en/simulations/build-a-fraction>

#### Fractions at Mathsisfun.com

A short illustrated tutorial about fractions. Includes an illustrative tool, and there are practice questions at the bottom of the page.

<https://www.mathsisfun.com/fractions.html>

#### Math Lines Fractions

Combine balls whose fractions add to one.

<https://www.mathnook.com/math2/math-lines-fractions.html>

#### Puzzle Pics Fractions

Reveal the mystery picture by dragging each puzzle piece to the number line that illustrates the fraction.

[https://www.mathplayground.com/puzzle\\_pics\\_fractions.html](https://www.mathplayground.com/puzzle_pics_fractions.html)

#### Animal Rescue: Fractions Number Line Game

Find and free the trapped animals by moving the arrow to the correct place on the number line.

<https://www.sheppardsoftware.com/math/fractions/animal-rescue/>

#### Clara Fraction's Ice Cream Shop

Convert improper fractions into mixed numbers, and scoop the right amount of ice cream flavors onto the cone for your customers.

<https://mrnuusbaum.com/clara-fraction-s-ice-cream-shop-online-game>

#### Write Mixed Numbers and Improper Fractions

Practice converting mixed numbers to improper fractions and vice versa with this interactive online exercise.

[https://www.khanacademy.org/math/cc-fourth-grade-math/imp-fractions-2/imp-mixed-numbers/e/converting\\_mixed\\_numbers\\_and\\_improper\\_fractions](https://www.khanacademy.org/math/cc-fourth-grade-math/imp-fractions-2/imp-mixed-numbers/e/converting_mixed_numbers_and_improper_fractions)

### ADDITION AND SUBTRACTION

#### Add and Subtract Fractions - Online Practice

Practice addition and subtraction of fractions with interactive customizable exercises.

<https://www.mathmammoth.com/practice/add-fractions>

#### Adding Like Fractions with Circle Models

Practice adding fractions with the help of a visual model.

<https://visualfractions.com/add-like-fractions-circle/>

#### Fractions Workshop

Choose "Add mixed fractions with like denominators" in order to practice adding mixed numbers.

<https://mrnuusbaum.com/fraction-workshop-online>

#### Add Mixed Numbers: 10-Question Quiz

Practice adding mixed numbers. Express the answer as a mixed fraction in lowest terms.

<https://www.thatquiz.org/tq-3/?-j301-11-p0>

## **Fraction Game**

Move the markers on the fraction number line from left to right according to the given fraction cards.

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Game/>

## **Four-Sum Fractions Board Game**

Practice adding and simplifying fractions with common denominators. Be the first player to score four in a row!

<https://www.learn-with-math-games.com/fraction-math-games.html>

## **Subtracting Mixed Numbers with Borrowing**

Perform subtraction calculations using borrowing with mixed number fractions.

<https://www.wisc-online.com/learn/formal-science/mathematics/abm701/subtracting-mixed-number-fractions-with-borro>

## **Subtracting Mixed Fractions Quiz**

Practice subtracting mixed fractions in this multiple-choice quiz. Drag and drop corresponding answers.

<https://www.fractions4kids.com/subtracting-mixed-fractions-quiz/>

## **Fruit Shoot Fractions Addition**

Click the fruit with the correct answer. To match the topics students learn in this section, choose adding 2 or 3 fractions with like denominators. You can also choose your mode (untimed or timed) and speed (slow versus fast fruit).

<https://www.sheppardsoftware.com/math/fractions/addition-game/>

## **Fraction Worksheets: Addition and Subtraction**

Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose “Like Fractions” for this level.

<https://www.homeschoolmath.net/worksheets/fraction.php>

## **EQUIVALENT FRACTIONS**

### **Equivalent Fraction Pairs**

Choose from five different activities that practice equivalent fractions.

[https://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Pairs.asp?Topic=12](https://www.transum.org/software/SW/Starter_of_the_day/Students/Pairs.asp?Topic=12)

### **Fresh Baked Fractions**

Practice equivalent fractions by clicking on a fraction that is not equal to the others.

<https://www.funbrain.com/games/fresh-baked-fractions>

### **Fractions: Equality**

Build equivalent fractions with different denominators. Match shapes and numbers to earn stars in the game.

[https://phet.colorado.edu/sims/html/fractions-equality/latest/fractions-equality\\_en.html](https://phet.colorado.edu/sims/html/fractions-equality/latest/fractions-equality_en.html)

### **Free Equivalent Fractions Worksheets**

Create custom-made worksheets for equivalent fractions that can either include pie images or not.

[https://www.homeschoolmath.net/worksheets/equivalent\\_fractions.php](https://www.homeschoolmath.net/worksheets/equivalent_fractions.php)

## **COMPARING FRACTIONS**

### **Visually Compare Fractions with Unlike Denominators**

Practice comparing fractions with different denominators with this interactive online quiz.

<https://cutt.ly/compare-fractions-unlike-denominators>

### **Compare Fractions with Different Numerators and Denominators**

Practice comparing unlike fractions with this online quiz from Khan Academy.

<https://cutt.ly/compare-fractions-different>

## Ordering Fractions

Practice ordering fractions from least to greatest in this interactive activity.

[https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-comparing-fractions/e/ordering\\_fractions](https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-comparing-fractions/e/ordering_fractions)

## Number Line Hunt

Place the fractions on the number line. Click a fraction then click its place on the number line.

[https://www.mathplayground.com/number\\_line\\_hunt.html](https://www.mathplayground.com/number_line_hunt.html)

## Number Climb

Click on the number spheres from smallest to largest value. Choose “Fractions”.

[https://www.mathplayground.com/number\\_climb.html](https://www.mathplayground.com/number_climb.html)

## Dirt Bike Comparing Fractions

Play tug of war while practicing comparing fractions in this interactive online game.

<https://www.arcademics.com/games/dirt-bike-comparing-fractions>

## MULTIPLYING FRACTIONS

### Multiplying Fractions with Circle Models

This page illustrates fraction multiplication with circle models.

<https://visualfractions.com/multiply-fractions-circle/>

### Multiply Fractions and Whole Numbers with Fraction Models

Practice multiplying fractions by whole numbers with the help of visual models in this short multiple-choice quiz.

<https://bit.ly/multiply-whole-numbers-fraction-models>

### Fractions by Wholes

A self-check exercise that practices multiplying fractions by whole numbers.

[https://www.transum.org/Maths/Puzzles/Jigsaw/Fraction\\_Whole.asp](https://www.transum.org/Maths/Puzzles/Jigsaw/Fraction_Whole.asp)

### Multiply Fractions by Whole Numbers

Practice multiplying fractions by whole numbers in this simple online exercise.

<https://www.mathgames.com/skill/4.67-multiply-fractions-by-whole-numbers>

### Multiplying Fractions Concentration

Practice multiplying fractions by whole numbers with this interactive online matching game.

<https://www.quia.com/cc/2740524.html>

## GENERAL

### Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated with a visual model. Also includes a couple of games.

<https://www.visualfractions.com/>

### Fractions App

An interactive app that can be used to represent, compare, and perform operations with fractions.

<https://www.mathlearningcenter.org/apps/fractions>

### Who Wants Pizza?

This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.

<https://bit.ly/pizza-fractions>

### Fractioncity

Make “fraction streets” to compare fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. Has directions for offline activity.

<https://teachnet.com/lessonplans/math/fraction-city/>

## Sample worksheet from

<https://www.mathmammoth.com>



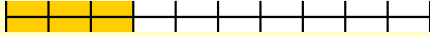
# One Whole and Its Fractional Parts

A fraction always relates to some kind of *one whole*. Study the examples below:



Let's say the one whole is this square. It is divided into 12 parts.

Each part is  $\frac{1}{12}$  of the whole. Also, we can write  $1 = \frac{12}{12}$ .

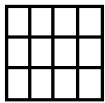
Maybe the one whole is this line, and  $\frac{3}{10}$  of it is colored. 

Maybe the one whole is Daddy's salary. To find  $\frac{5}{6}$  of it, imagine dividing the salary into 6 parts, and taking five of those parts. All six parts form the one whole, or  $\frac{6}{6} = 1$

$\frac{7}{12}$  The top number is the **numerator**. It *numerates* or counts *how many pieces* there are.  
 The bottom number is the **denominator**. It *denominates* or *names* what kind of parts they are.

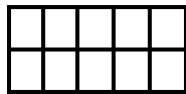
1. Color parts. Write the colored part *and* the white (uncolored) part as a fraction.

a. Color 1 part.



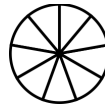
$\frac{1}{12}$  and —

b. Color 5 parts.



and

c. Color 8 parts.



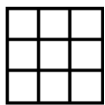
and

d. Color 3 parts.



and

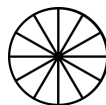
2. Color and write one whole as a fraction.



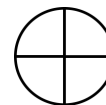
a.  $1 = \text{---}$



b.  $1 = \text{---}$



c.  $1 = \text{---}$



d.  $1 = \text{---}$



e.  $1 = \text{---}$

3. Solve.

a. The Jacksons ate  $\frac{3}{4}$  of the pie.

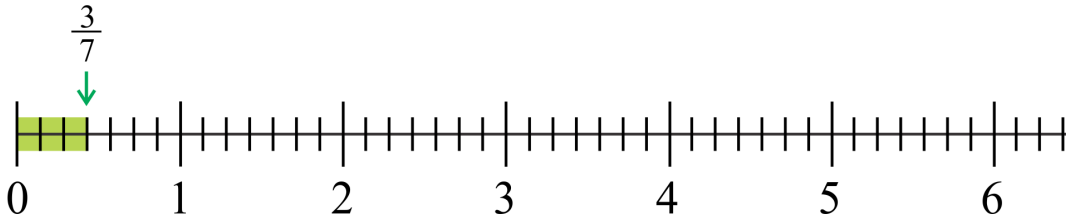
How much is left?

b. Jerry ate  $\frac{1}{6}$  of the pizza.

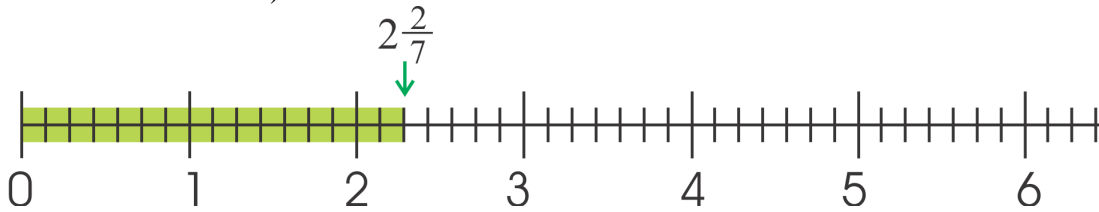
How much is left?

c. Five boys shared a chocolate bar equally. Each one got — of the bar.

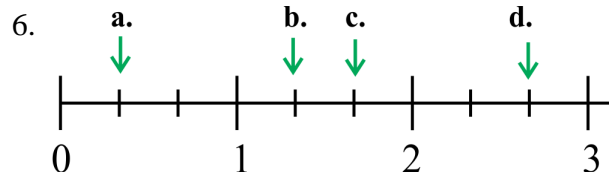
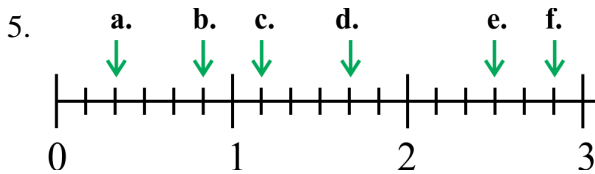
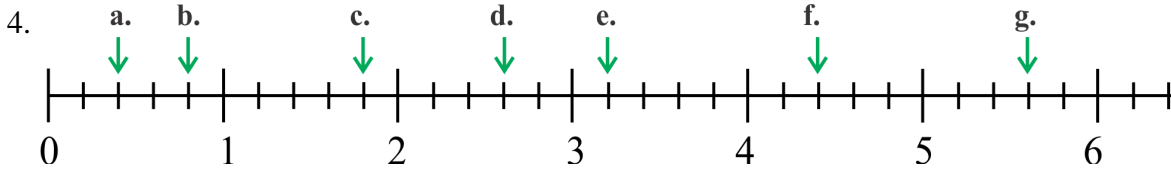
To show  $\frac{3}{7}$  on a number line, each whole-number interval (from 0 to 1, from 1 to 2, from 2 to 3, and so on) is divided into seven parts. Three of those parts are colored to show  $\frac{3}{7}$ .



In a **mixed number**, we have a whole number and a fraction. The number line below shows  $2\frac{2}{7}$  (two and two sevenths).



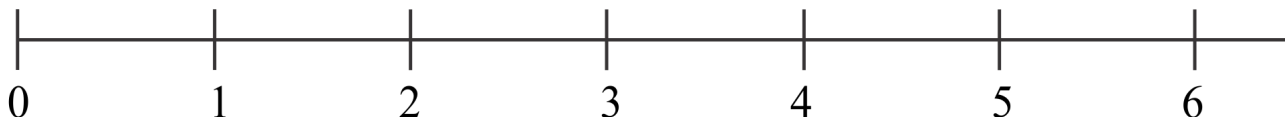
In problems 4 - 6, write the fractions and mixed numbers that the arrows mark on the number line.



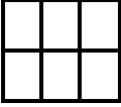
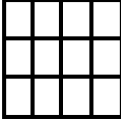
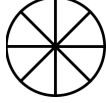
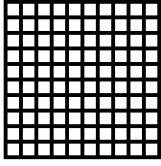
7. Mark the mixed numbers on the number line:

- a.**  $1\frac{2}{4}$    **b.**  $\frac{3}{4}$    **c.**  $4\frac{1}{4}$    **d.**  $5\frac{1}{2}$    **e.**  $3\frac{1}{4}$    **f.**  $2\frac{3}{4}$

*Hint: First divide each whole-number interval into four parts (using three tick marks).*



8. Color. Then write an addition, adding the colored and white parts. Notice what sum you get.

<p><b>a.</b> Color 1 part.</p>  <p><math>\frac{1}{6} + \text{---} = 1</math></p>	<p><b>b.</b> Color 10 parts.</p> 	<p><b>c.</b> Color 3 parts.</p> 	<p><b>d.</b> Color 15 parts.</p> 
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9. Find what fraction is missing from one whole.

**a.**  $\frac{3}{4} + \text{---} = 1$      
 **b.**  $\frac{6}{7} + \text{---} = 1$      
 **c.**  $\frac{1}{8} + \text{---} = 1$      
 **d.**  $\frac{11}{12} + \text{---} = 1$

10. **a.** Mary drank  $\frac{1}{4}$  liter of juice from a 1-liter pitcher, and her brother drank another  $\frac{1}{4}$  liter.  
How much juice is left in the pitcher?

**b.** A loaf of bread was cut into 20 slices. Jack and John ate three slices each.  
What fractional part of the bread is left?

11. Let's review how to find a fractional part using division.

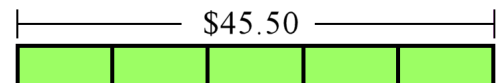
**a.** Remember division? Find  $\frac{1}{10}$  of 90 km.

Then find  $\frac{4}{10}$  of 90 km.

**b.** A restaurant bill was \$45.50. It was divided so that Cindy paid  $\frac{2}{5}$  of it and Sandy paid  $\frac{3}{5}$  of it.

How many dollars did Cindy pay?

How many dollars did Sandy pay?



**c.** Dad used  $\frac{2}{9}$  of his \$2,700 paycheck.

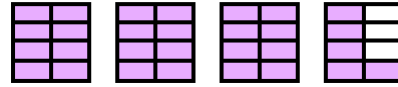
What fractional part is left of his paycheck?

How many dollars are left of his paycheck?

# Mixed Numbers

**Mixed numbers have two parts:** a whole-number part and a fractional part.

This picture illustrates  $3\frac{5}{8}$ : *three and five eighths*.



Notice: the colored portion is  $3\frac{5}{8}$ . The uncolored part is  $\frac{3}{8}$ .

If we add the colored and uncolored parts, we get four wholes:  $3\frac{5}{8} + \frac{3}{8} = 4$ .

1. Write the mixed numbers these pictures illustrate.

<p>a.</p>	<p>b.</p>	<p>c.</p>
<p>d.</p>	<p>e.</p>	<p>f.</p>

2. Write an addition sentence, adding what is colored and what is not. Look at the example.

<p>a.</p> <p><math>2\frac{2}{4} + \frac{2}{4} = 3</math></p>	<p>b.</p>	<p>c.</p>
<p>d.</p>	<p>e.</p>	<p>f.</p>

3. How much is missing from the next whole number?

<p>a.</p> <p><math>1\frac{1}{4} + \frac{\quad}{\quad} = 2</math></p>	<p>b.</p> <p><math>3\frac{2}{10} + \frac{\quad}{\quad} = 4</math></p>	<p>c.</p> <p><math>8\frac{4}{9} + \frac{\quad}{\quad} = 9</math></p>	<p>d.</p> <p><math>5\frac{1}{8} + \frac{\quad}{\quad} = 6</math></p>
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# Adding Mixed Numbers

**Example 1.**

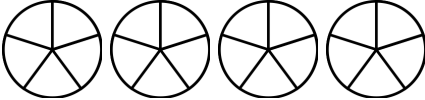
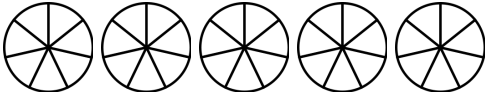
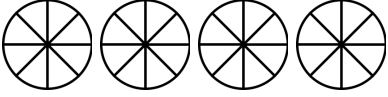
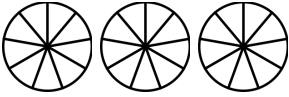
Add the whole numbers and the fractions separately.

Since  $\frac{10}{8}$  is  $\frac{8}{8} + \frac{2}{8}$ , or 1 and  $\frac{2}{8}$ ,

the final answer becomes  $7 \frac{2}{8}$ .

$$\begin{aligned} 2 \frac{3}{8} + 4 \frac{7}{8} &= 6 \frac{10}{8} \\ &= 6 \frac{8}{8} + \frac{2}{8} \\ &= 7 \frac{2}{8} \end{aligned}$$

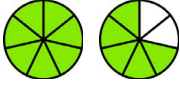

1. Add the mixed numbers. You can shade parts to help.

 <p><b>a.</b> <math>1 \frac{3}{5} + 2 \frac{2}{5} =</math></p>	 <p><b>b.</b> <math>1 \frac{3}{7} + 2 \frac{6}{7} =</math></p>
 <p><b>c.</b> <math>1 \frac{3}{8} + 1 \frac{6}{8} =</math></p>	 <p><b>d.</b> <math>\frac{8}{9} + 1 \frac{5}{9} =</math></p>

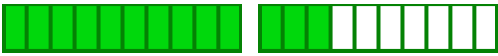




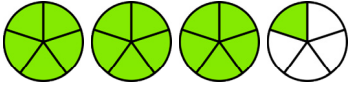




2. Add the mixed numbers.

<p><b>a.</b> <math>1 \frac{2}{5} + 3 \frac{3}{5} =</math></p>	<p><b>b.</b> <math>4 \frac{2}{6} + 2 \frac{5}{6} =</math></p>
<p><b>c.</b> <math>5 \frac{2}{4} + 7 \frac{3}{4} =</math></p>	<p><b>d.</b> <math>1 \frac{3}{8} + 8 \frac{7}{8} =</math></p>
<p><b>e.</b> <math>7 \frac{1}{6} + 20 \frac{1}{6} =</math></p>	<p><b>f.</b> <math>8 \frac{9}{10} + 3 \frac{3}{10} =</math></p>

3. Pretend you are the “teacher” and find the errors in the these students’ work! Then fix them.

<p><b>a. Emma:</b> </p> $1\frac{5}{7} = \frac{2}{7} + 1\frac{2}{7} + \frac{2}{7}$ $1\frac{5}{7} = \frac{10}{7} + \frac{2}{7}$	<p><b>b. Peter:</b> </p> $2\frac{1}{3} = \frac{2}{3} + \frac{2}{3} + \frac{1}{3} + \frac{1}{3}$ $2\frac{1}{3} = \frac{5}{3} + \frac{3}{3}$
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4. Write each mixed number as an addition in different ways.

<p><b>a.</b> </p> <p> =</p> <p> =</p> <p> =</p> <p> =</p>	<p><b>b.</b> </p> <p> =</p> <p> =</p> <p> =</p> <p> =</p>
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5. Solve.

**a.** A recipe calls for  $1\frac{1}{2}$  cups of wheat flour,  $\frac{1}{2}$  cup of rye flour, and  $\frac{1}{2}$  cup of oat flour.  
How much flour in total does the recipe use?

**b.** A movie lasted  $1\frac{3}{4}$  hours and a meal afterwards took  $1\frac{1}{4}$  hours. How much time in total did these take?

**c.** Jack drank  $1\frac{1}{4}$  cups of water and  $\frac{3}{4}$  cups of juice.  
How much liquid did he drink in total?

6. The sides of a rectangle are  $2\frac{1}{4}$  inches and  $3\frac{1}{4}$  inches long. Draw a sketch. What is its perimeter?

7. Each side of a triangle is  $2\frac{3}{8}$  inches. What is its perimeter?

8. Double the cake recipe.

A birthday cake

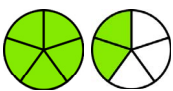
4 eggs  
 $\frac{3}{4}$  cup sugar  
 $1\frac{1}{4}$  cup flour  
 $1\frac{1}{2}$  tsp baking powder  
 1 cup whipped cream  
 sliced fruit

A birthday cake

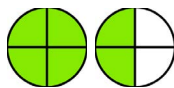
\_\_\_\_\_ eggs  
 \_\_\_\_\_ cup sugar  
 \_\_\_\_\_ cup flour  
 \_\_\_\_\_ tsp baking powder  
 \_\_\_\_\_ cup whipped cream  
 sliced fruit

**Puzzle Corner**

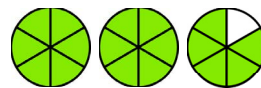
The picture shows the first addend. Draw more pies to figure out how much is missing from the additions.



**a.**  $1\frac{2}{5} + \quad = 2\frac{3}{5}$



**b.**  $1\frac{2}{4} + \quad = 3\frac{1}{4}$

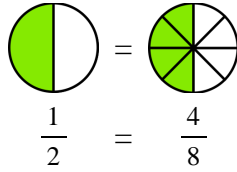


**c.**  $2\frac{5}{6} + \quad = 4\frac{2}{6}$

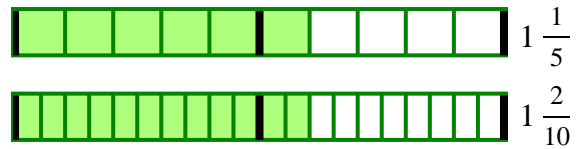


# Equivalent Fractions

If you eat half of a pizza, or if you eat  $\frac{4}{8}$  of a pizza, you have eaten the same amount.



$\frac{1}{2}$  and  $\frac{4}{8}$  are **equivalent fractions**.



The two fraction strips show an equal amount. So, we can write an equal sign between the

two mixed numbers:  $1 \frac{1}{5} = 1 \frac{2}{10}$ .

1. Color the first fraction. Shade the same amount of pie in the second picture. Write the second fraction.

 a. $\frac{1}{2} =$	 b. $\frac{3}{4} =$	 c. $\frac{6}{10} =$	 d. $\frac{8}{12} =$
 e. $\frac{1}{3} =$	 f. $1 \frac{2}{3} =$	 g. $1 \frac{10}{12} =$	

2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.

	a. $\frac{3}{3} =$	b. $\frac{4}{3} =$	c. $\frac{7}{3} =$
	d. $2 \frac{1}{3} =$	e. $1 \frac{2}{3} =$	f. $2 \frac{2}{3} =$

3. Mark the equivalent fractions on the number lines.

 a. $\frac{4}{5} =$	 b. $\frac{3}{9} =$
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# Chapter 8: Decimals

## Introduction

In fourth grade, students learn about decimal numbers that have one or two decimal digits, and they learn to add and subtract them. It is important to grasp these simple topics well because we are laying a groundwork for fifth and sixth grades where decimal operations take “center stage.”

The focus is, first of all, on understanding that decimals are simply fractions with a denominator of 10 or 100. Then with that in mind, we study comparing, adding, and subtracting them.

Take note of this common misconception that students have. Many students add  $0.5 + 0.9 = 0.14$ . The correct way to view  $0.5 + 0.9$  is as 5 tenths plus 9 tenths, which is 14 tenths = 1.4.

An example of another misconception is when a student adds  $0.5 + 0.11 = 0.16$ . This student is thinking of the decimal parts as if they were “whole numbers” and adding  $5 + 11 = 16$ . To solve  $0.5 + 0.11$  correctly, students can rewrite 0.5 as 0.50, and then the problem becomes  $0.50 + 0.11 = 0.61$ .

In the lesson *Using Decimals with Measuring Units*, students encounter decimals in connection with metric units, such as 0.1 km or 2.4 kg, and they also convert between the units, such as writing 0.5 km as 500 m. This topic will be studied further in 5th grade.

### The Lessons in Chapter 8

	page	span
Decimal Numbers—Tenths .....	177	2 pages
Adding and Subtracting with Tenths .....	179	2 pages
Two Decimal Digits—Hundredths .....	181	4 pages
Add and Subtract Decimals in Columns .....	185	3 pages
Add and Subtract Decimals Mentally .....	188	4 pages
Using Decimals with Measuring Units .....	192	2 pages
Mixed Review Chapter 8 .....	194	2 pages
Review Chapter 8 .....	196	2 pages

### Helpful Resources on the Internet

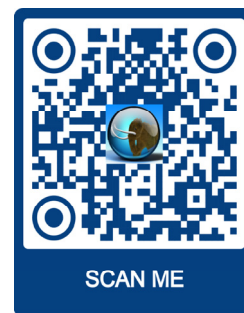
You can also access this list of links at <https://l.mathmammoth.com/gr4ch8>

**DISCLAIMER:** We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.

#### Fractions & Decimals Matching Game

Practice converting fractions to decimals while also uncovering a hidden picture in this fun matching game!

<https://www.mathmammoth.com/practice/fractions-decimals>



Sample worksheet from  
<https://www.mathmammoth.com>

### **Decimal Place Value - Hundredths**

Practice identifying numbers that have two decimal digits with this interactive multiple-choice quiz.

[https://docs.google.com/forms/d/e/1FAIpQLSe-vn9Cs6S2cT1\\_nBmM7uZsiEFwj1Lh3CKuBQZxES\\_7NtSTWw/viewform](https://docs.google.com/forms/d/e/1FAIpQLSe-vn9Cs6S2cT1_nBmM7uZsiEFwj1Lh3CKuBQZxES_7NtSTWw/viewform)

### **Modeling Decimals (Area and Grid Models)**

An interactive “gizmo” for modeling decimals in a grid or on a number line. It is by subscription, but you can try the gizmo for 5 minutes for free.

<https://www.explorelarning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007>

### **Decimals Quiz**

Test your knowledge of decimals with this interactive online quiz.

<https://www.khanacademy.org/math/cc-fourth-grade-math/imp-decimals/quiz/imp-decimals-quiz-2?modal=1>

### **Printable Math Puzzles**

This page has several brain teasers and puzzles which will help the student apply and practice their math skills to solve a range of challenges and number problems.

<https://www.math-salamanders.com/printable-math-puzzles.html>

### **Write Decimals as Fractions**

Practice writing decimals as fractions in this interactive online activity.

<https://cutt.ly/write-decimals-fractions>

### **Fraction Lines**

Sort the decimal numbers in order from smallest to largest.

[https://www.transum.org/Software/sw/Starter\\_of\\_the\\_day/Students/Fraction\\_line/Default.asp?Level=3](https://www.transum.org/Software/sw/Starter_of_the_day/Students/Fraction_line/Default.asp?Level=3)

### **Reading Scales**

Practice reading a linear scales showing decimals in this self-check quiz.

[https://www.transum.org/Maths/Activity/Reading\\_Scales/Default.asp?Level=3](https://www.transum.org/Maths/Activity/Reading_Scales/Default.asp?Level=3)

### **Fraction/Decimal Worksheets**

Change fractions to decimal numbers or decimal numbers to fractions.

<https://www.homeschoolmath.net/worksheets/fraction-decimal.php>

### **Fraction Snake Game**

Arrange the numbers on the snake in order from the largest on the head to the smallest at the tail.

[https://www.transum.org/software/SW/fracorder/fraction\\_order.asp](https://www.transum.org/software/SW/fracorder/fraction_order.asp)

### **Decimal Subtraction Game**

Match each subtraction problem to its correct answer.

<https://games.forkids.education/decimal-subtraction-game/>

### **Number Conundrum Decimals**

Complete the number puzzle using decimal numbers. The number in each block is the sum of the two numbers directly underneath.

[https://www.mathplayground.com/number\\_conundrum\\_decimals.html](https://www.mathplayground.com/number_conundrum_decimals.html)

### **Decimals Quiz**

Practice adding and subtracting decimals in this 10-question quiz.

<https://www.thatquiz.org/tq-3/?-j163-15-p0>

### **Decimal Subtraction Mystery Picture**

Match the decimal subtraction problems with the correct answers to uncover a hidden picture.

<https://www.mathmammoth.com/practice/mystery-picture-decimals#digits=2&mode=subtraction>

## **Sample worksheet from**

<https://www.mathmammoth.com>

### **Decimal Mania - Addition and subtraction**

Practice decimal addition and subtraction with this interactive exercise.

<https://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/addsubdec.shtml>

### **Adding Decimals: Hundredths**

Practice adding numbers that have two decimal digits in this interactive online activity.

[https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-add-decimals/e/adding\\_decimals](https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-add-decimals/e/adding_decimals)

### **Sum to One**

Choose from five different activities that practice mental addition of decimals.

[https://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Pairs.asp?Topic=22](https://www.transum.org/software/SW/Starter_of_the_day/Students/Pairs.asp?Topic=22)

### **Get to the (Decimal) Point Addition and Subtraction card games (pp. 60-63 of the PDF file)**

This is a card game with four different variations that practice decimal addition and subtraction.

<https://cutt.ly/math-card-games>

### **Decimal Plus**

Practice mental and written methods for adding and subtracting decimal numbers.

<https://www.transum.org/Maths/Activity/Decimals/Add.asp>

### **Decimal Squares Speedway**

Practice decimal addition with this interactive game that can be played against the computer or against another person.

[https://decimalsquares.com/dsGames/Decimal\\_Speedway/index.htm#](https://decimalsquares.com/dsGames/Decimal_Speedway/index.htm#)

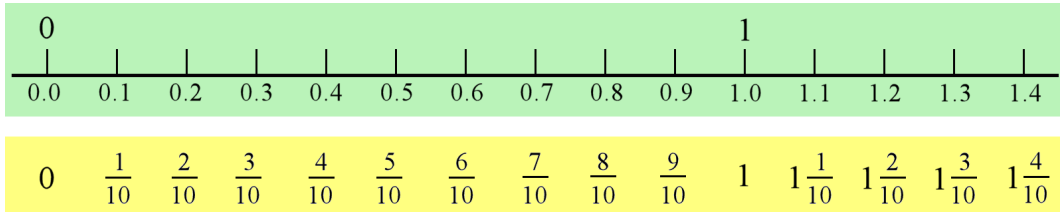
### **Convert units (metrics)**

Practice converting between metric units of measurement in this interactive online exercise.

<https://www.khanacademy.org/math/cc-fifth-grade-math/imp-measurement-and-data-3/imp-unit-conversion/e/converting-units>

## Decimal Numbers—Tenths

The number line between 0 and 1 is divided into ten parts. Each of these ten parts is  $\frac{1}{10}$ , a **tenth**.



Under the tick marks, you see **decimal numbers** such as 0.1, 0.2, 0.3, and so on.

These are the same numbers as the fractions  $\frac{1}{10}$ ,  $\frac{2}{10}$ ,  $\frac{3}{10}$ , and so on.

The digit right after the decimal point (such as the digit 3 in 0.3) tells us **how many tenths** the number has. That digit is in the tenths place. So, 0.3 mean—and is read as—three tenths.

0.6 means six tenths, or  $\frac{6}{10}$ .

1.5 means 1 whole and 5 tenths, or  $1\frac{5}{10}$ .

**Note:**  $\frac{1}{8}$  is *not* 0.8. Instead, 0.8 is eight tenths, or  $\frac{8}{10}$ .  
The denominator is always 10!

1. Write the fractions as decimals and vice versa.

a. $\frac{7}{10}$	b. $2\frac{4}{10}$	c. $10\frac{9}{10}$	d. 0.9	e. 29.3
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2. Write the decimal and the fraction that each picture shows.

a.	b.	c.
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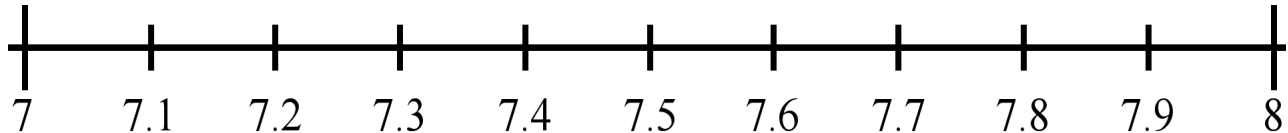
3. Shade parts to show the decimals.

a. 0.4	c. 1.6	d. 2.8
b. 0.1		

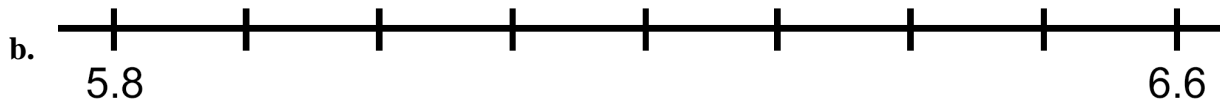
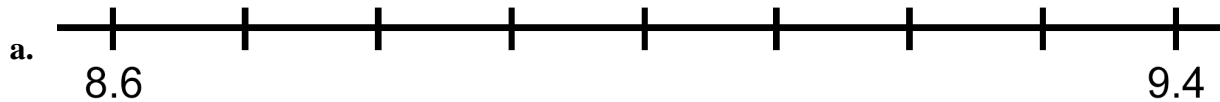
**“Decimal”** comes from the Latin word *decem*, which simply means **“ten.”** The way we write numbers is a **decimal number system**, because it is based on number ten: we use ten different digits (from 0 to 9) and write digits in places such as the ones place, tens place, hundreds place, and so on, each of those places having a value that is ten times the value of the previous place.

In common language, the word “decimal number” has come to mean numbers which have digits after the decimal point, such as 5.8 or 9.302. In reality, any number within the decimal number system could be termed as a decimal number, including whole numbers such as 12 or 381.

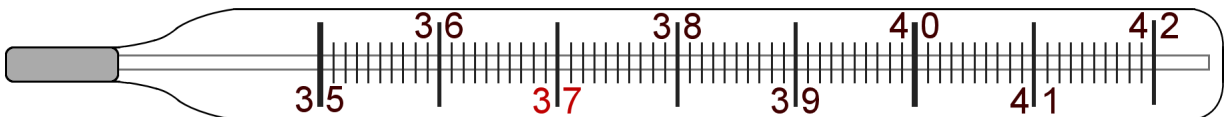
4. Write a mixed number under each decimal number.



5. Label the tick marks with decimal numbers.



6. a. Mark these temperatures with dots on the thermometer:  
37.4°C, 36.2°C, 38.7°C, 41.8°C, 40.5°C



b. Which temperatures would indicate a fever?

7. Compare. Write  $<$ ,  $>$ , or  $=$  between the numbers.

a.  $0.5 \square 0.9$

b.  $1.3 \square 0.3$

c.  $5.1 \square 4.9$

d.  $0.4 \square \frac{1}{2}$

e.  $16.0 \square 16$

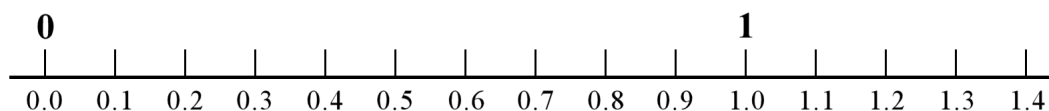
8. Write in order from the smallest to the largest number:

1.2   0.9   2.6   0.1    $2\frac{1}{2}$    2.3   3.0    $\frac{1}{2}$

## Adding and Subtracting with Tenths

<p>You already know how to add or subtract decimals that have tenths, such as <math>0.8 + 0.5</math>. They are just fractions with a denominator of 10.</p> <p>Compare the two additions in each box. One of them is written with decimals and the other with fractions.</p>	$0.1 + 0.5 = 0.6$  $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$  $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
<p>There is one tricky thing: <math>0.6 + 0.7</math> is <b><u>NOT</u></b> 0.13!</p> <p>To see why, add the corresponding fractions. Notice that six-tenths and seven-tenths makes thirteen-tenths, which is more than one!</p>	$0.6 + 0.7 = 1.3$  $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$  $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each “number-line jump”.



- a. You are at 0.7, and you jump *five tenths* to the right. \_\_\_\_\_
- b. You are at 0.6, and you jump *eight tenths* to the right. \_\_\_\_\_
- c. You are at 1.1, and you jump *eight tenths* to the left. \_\_\_\_\_
- d. You are at 1.3, and you jump *four tenths* to the left. \_\_\_\_\_
- e. You are at 0.2, and you jump *eleven tenths* to the right. \_\_\_\_\_

2. Solve the fraction additions, and then write them using decimals.

<p>a. <math>\frac{2}{10} + \frac{7}{10} =</math></p> <p><math>0.2 +</math></p>	<p>b. <math>\frac{5}{10} + \frac{6}{10} =</math></p>	<p>c. <math>\frac{9}{10} + \frac{8}{10} =</math></p>
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3. Add or subtract.

<b>a.</b>	<b>b.</b>	<b>c.</b>	<b>d.</b>
$0.9 + 0.2 =$ _____	$0.5 + 0.7 =$ _____	$0.8 + 0.7 =$ _____	$1.8 - 0.9 =$ _____
$1.9 + 0.2 =$ _____	$3.5 + 0.7 =$ _____	$0.8 + 2.7 =$ _____	$5.8 - 0.9 =$ _____



4. Calculate.

<b>a.</b>	<b>b.</b>	<b>c.</b>	<b>d.</b>
$2.3 + 0.9 = \underline{\hspace{2cm}}$	$1.5 + 0.7 = \underline{\hspace{2cm}}$	$6.6 - 0.5 = \underline{\hspace{2cm}}$	$4.7 - 1.7 = \underline{\hspace{2cm}}$

5. Write the numbers.

- a. 3 tenths, 5 ones  
 b. 7 tens, 8 ones, 4 tenths  
 c. 4 tenths, 3 ones, 6 tens

T	O	.	te
4	7	.	5

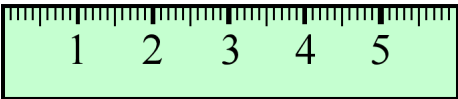
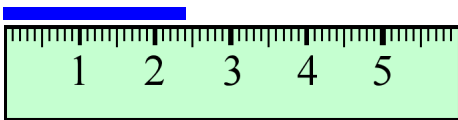
In this place value chart, “T” means tens, “O” means ones, and “te” means tenths.

We can see that the number 47.5 has 4 tens, 7 ones, and 5 tenths.

6. Continue the patterns by adding or subtracting the same number repeatedly.

<b>a. 0.1</b>	<b>b. 1.1</b>	<b>c. 2.5</b>	<b>d. 3.6</b>
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$
$+ 0.2 = \underline{\hspace{2cm}}$	$+ 0.5 = \underline{\hspace{2cm}}$	$+ 0.3 = \underline{\hspace{2cm}}$	$- 0.4 = \underline{\hspace{2cm}}$

7. Remember: **1 millimeter is one-tenth of a centimeter.** Or,  $1 \text{ mm} = 0.1 \text{ cm}$ .

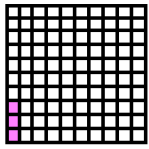
<p><b>a.</b> Draw a line that is 4.7 cm long.</p> 	<p><b>b.</b> Measure the line in centimeters. Use a decimal.</p> 
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8. In (a) and (b), convert. In (c), add and give your answer in centimeters.

- a.  $0.5 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$       b.  $7 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$       c.  $5 \text{ mm} + 0.9 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$   
 $1.2 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$        $35 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$        $4 \text{ cm} + 3.4 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$

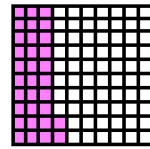
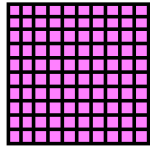
9. The two sides of a rectangle measure 6.5 cm and 3.6 cm.  
 Draw the rectangle on blank paper. What is its perimeter?

## Two Decimal Digits—Hundredths



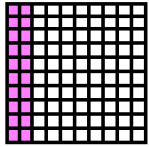
This is 3 hundredths ( $3/100$ ).  
As a decimal, we write **0.03**.

Read 0.03 as “three hundredths.”



This is 1 and 32 hundredths  
( $1 \frac{32}{100}$ ). As a decimal,  
we write **1.32**.

Read 1.32 as “one and 32 hundredths.”

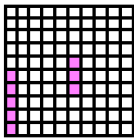


This is 20 hundredths ( $20/100$ ). As a decimal, we write it as **0.20**.

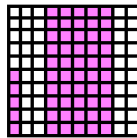
It is *also* two tenths ( $2/10$  or  $0.2$ ), because it is two columns, and each column is one-tenth of the whole. So,  $0.20 = 0.2$ , or 20 hundredths equals 2 tenths.

**The two decimal digits after the decimal point indicate *hundredths*.**

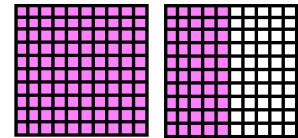
1. Write the number that each picture illustrates as a decimal *and* as a fraction or mixed number. Then read the decimals aloud.



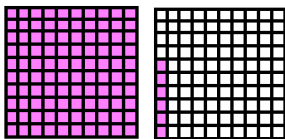
a. \_\_\_\_\_ =  $\frac{\square}{\square}$



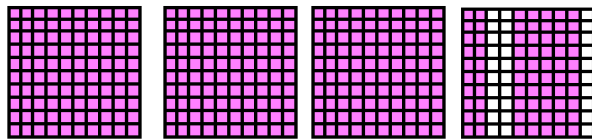
b. \_\_\_\_\_ =  $\frac{\square}{\square}$



c. \_\_\_\_\_ =  $\square \frac{\square}{\square}$

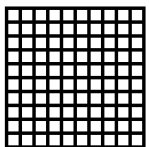


d. \_\_\_\_\_ =  $\square \frac{\square}{\square}$

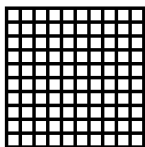


e. \_\_\_\_\_ =  $\square \frac{\square}{\square}$

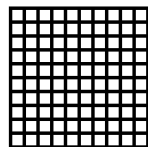
2. Color to illustrate the decimals. Then write them as fractions. Read the decimals aloud.



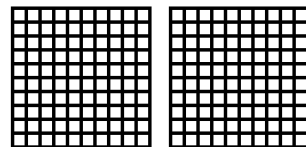
a.  $0.52 =$



b.  $0.7 =$

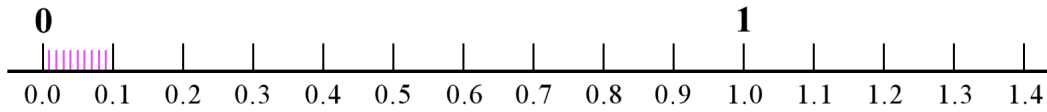


c.  $0.09 =$



d.  $1.08 =$

You have seen this number line already. In it, the distance from 0.0 to 0.1 is *one tenth*. Now we have also drawn nine tiny lines between 0.0 and 0.1, dividing that distance **into TEN new parts**.

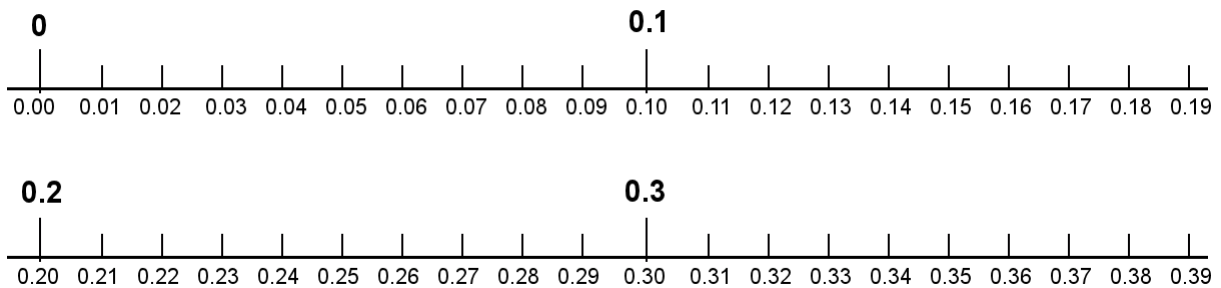


Now, *draw* nine tiny lines between 0.2 and 0.3, dividing that distance into TEN new parts.

If this process was repeated between 0.3 and 0.4, between 0.4 and 0.5, and so on, into how many parts in total would the number line from 0 to 1 be divided? \_\_\_\_\_ parts

These new parts are therefore **hundredth parts**, or **hundredths**.

The number line below *zooms in* to the previous number line. The distance from 0 to 0.1 is divided into ten parts, the distance from 0.1 to 0.2 is divided into ten parts also, and so on. Each small interval is **one hundredth**.



The numbers at the tick marks have two decimal digits (two digits after the decimal point).

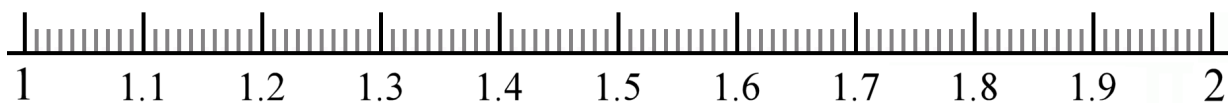
The number 0.28 is read as *twenty-eight hundredths* and is the same as  $\frac{28}{100}$ .

The number 0.06 is read as *six hundredths* and is the same as  $\frac{6}{100}$ .

The number 2.34 is read as *two and thirty-four hundredths* and is the same as  $2\frac{34}{100}$ .

3. Mark these decimals with dots on the number line below:

1.55   1.11   1.28   1.39   1.88   1.02   1.67   1.99   1.74   1.43   1.90   1.06   1.20



4. Fill in the missing decimals under the tick marks.

