Solving Equations

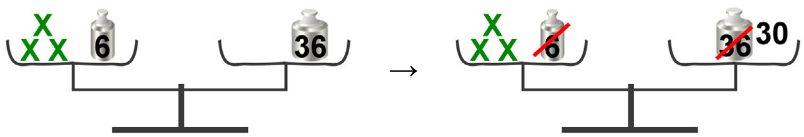
Solving equations is like a "game." The "goal" of the game is to leave the unknown (such as x) **alone** on one side of the equation, or **isolate** it, so that we have x = (something) or (something) = x.

The allowed "moves" in the game are these:

- You can *add* the same number to both sides of the equation;
- You can *subtract* the same number from both sides of the equation;
- You can *multiply* both sides of the equation by the same number;
- You can *divide* both sides of the equation by the same number.

Think of the balance: if you **add** the same thing to **both** pans of the balance, both sides will still weigh the same (though more than before)! Or, if you **take away** the same thing from **both** pans of the balance, both sides will STILL weigh the same (though less than before)!

Example 1. When we remove 6 from both sides of the pan balance below, the balance WILL stay balanced! The equation $3x + 6 = 36$ will "lose" 6 from both sides, and become $3x = 30$.



$3x + 6 = 36$ $\qquad$ $3x = 30$

Whatever you do (add, subtract, multiply, divide) to one side of the equation, you do to the other side as well. That preserves the equality of the two sides!

Study these examples carefully. They not only show how these simple equations are solved, but also illustrate *how to write down* the solution process—in two different ways. It is something you need to learn.

Example 2. The left side has the sum of x and 19. To make x appear alone, we *subtract 19 from both sides*. Notice how this is written in the solution. Now, on the left side, + 19 − 19 equals zero, so, we have only x left. On the right side, we simply calculate 454 − 19 = 435.

$$\begin{aligned} x + 19 &= 454 \\ \underline{-\ 19} \quad &\quad \underline{-\ 19} \\ x &= 435 \end{aligned}$$

Lastly, we need to **check** our solution. To do that, write the solution 435 in place of x in the *original* equation, and check that it is a true equation: **Is 435 + 19 really 454?** Yes, it is.

Example 3. This time, 76 is subtracted from x, so *add* 76 to both sides. Here is another way to mark what is going to be done to both sides of the equation: write it in the right margin.

On the left side, − 76 and + 76 cancel each other (equaling zero). On the right side, we calculate 180 + 76. We get 256.

$$\begin{aligned} x - 76 &= 180 \qquad \Big|\ +\ 76 \\ x - 76 + 76 &= 180 + 76 \\ x &= 256 \end{aligned}$$

Check: Does 256 − 76 really equal 180? Yes, it does.

Example 4. To leave y alone on the left side, we **add** 72 to both sides.

$$\begin{aligned} y - 72 &= 489 \\ \underline{+\ 72} \quad &\quad \underline{+72} \\ y &= 561 \end{aligned}$$

Check: Is 561 − 72 really 489? Yes, it is.

Example 5. To "undo" $48 + w$, we subtract 48 from both sides.

$$\begin{aligned} 48 + w &= 91 \qquad \Big|\ -\ 48 \\ 48 + w - 48 &= 91 - 48 \\ w &= 43 \end{aligned}$$

Check: Is 48 + 43 really 91? Yes, it is.

1. Solve these one-step equations. Look at the examples on the previous page, and write the steps to the solution. You can choose which way you will write them down (under the equation or in the margin).

a. $54 + x = 990$ $=$ $=$	**b.** $x + 5.6 = 12.9$ $=$ $=$
c. $x - 120 = 137$ $=$ $=$	**d.** $w - 98 = 89$ $=$ $=$
e. $156 + s = 1{,}082$ $=$ $=$	**f.** $t + 77 = 208$ $=$ $=$

Example 6. Here we will first simplify what is on the right side of the equation.

Simplify 45 + 18.	$35 + x = 45 + 18$	
Subtract 35 from both sides.	$35 + x = 63$	**− 35**
35 and − 35 cancel each other. (This step could be omitted.)	$35 + x - 35 = 63 - 35$	
The final result.	$x = 28$	

Check: Is 35 + 28 really equal to 45 + 18? Yes, it is. So, the solution $x = 28$ is correct.

Example 7. You can always switch the two sides of an equation. This is especially handy if the unknown is on the right side at first.

$$460 = x + 98$$
$$x + 98 = 460 \quad \Big| -98$$
$$x = 362$$

Do you have to do this? No, you don't, but it can be quite helpful at times.

2. Solve.

a. $y - 26 = 36 + 9$ $=$	**b.** $z - 220 = 3 \cdot 100$ $=$	**c.** $200 + x = 430 + 80$ $=$
d. $2.4 + 9.1 = 7 + z$ $=$	**e.** $4 \cdot 7 + 30 = s - 86$ $=$	**f.** $8 + x + 2.5 = 20 - 8.2$ $=$

$8x = 120$ $\frac{8x}{\mathbf{8}} = \frac{120}{\mathbf{8}}$ $x = 15$	**Example 8.** Here, x is multiplied by 8. To "undo", divide both sides by 8. On the left side, the 8 in the denominator cancels the 8 in the numerator. On the right side, calculate 120 ÷ 8. We get 15 as the root. Lastly, check: is 8 · 15 really 120?
Example 9. Here, x is divided by 9. To "undo" that and leave x by itself, multiply both sides by 9. On the left, the 9s in the numerator and in the denominator cancel each other. On the right, calculate 54 · 9. This is the final answer. To check, divide that by 9. Do you get 54?	$\frac{x}{9} = 54$	· 9 $\frac{x}{9} \cdot 9 = 54 \cdot 9$ $x = 486$

3. Solve these one-step equations. Write the solution steps in a manner similar to the examples above.

a. $5x = 350$ = =	**b.** $10x = 17$ = =	**c.** $7a = 2.8$ = =
d. $\frac{x}{51} = 4$ = =	**e.** $\frac{x}{9} = 60$ = =	**f.** $\frac{x}{100} = 1.2$ = =

4. Solve. In these, the unknown may be on the right side, and/or you may need to first simplify something.

a. $y \div 400 = 6 + 2$ = = =	**b.** $6 \cdot 9 = \frac{x}{20}$ = = =	**c.** $8x = 501 + 59$ = = =

5. **a.** Solve the equation $x = 2x$ in the set {5, 10, 15, 20}.

b. Can you find a solution to the equation outside of that set?
Hint: try some easy numbers.

6. Solve these equations. Think of equivalent fractions!

a. $\frac{x}{18} = \frac{5}{6}$	**b.** $\frac{14}{24} = \frac{7}{y}$	**c.** $\frac{5}{8} = \frac{z}{56}$

Example 10. This equation has several like terms ($3x$ and $4x$). But to isolate x, we need to have a single term with x, not several.

So, first we simplify $3x + 4x$ on the left side.	$3x + 4x = 35$	
Now we divide both sides by 7.	$7x = 35$	$\div 7$
Here is the final solution.	$x = 5$	

Check: is $3 \cdot 5 + 4 \cdot 5$ equal to 35? Yes, it is, so the solution checks.

7. Solve these equations.

a. $2y + 5y = 49$ $=$	**b.** $10x - 8x = 42$ $=$	**c.** $7a + 2a - 5a = 52$ $=$
d. $2x + 3x = 29 - 14$ $=$	**e.** $7c - c = 3 \cdot 80$ $=$	**f.** $14x - 6x + 2x = 5 \cdot 40$ $=$

Two-step equations (*optional*). Remember, the goal in solving equations is to have the term with x by itself on one side of the equation. Study the examples of equations that are solved in two steps.

Example 11.

$4x + 17 = 81$	$- 17$
$4x = 64$	$\div 4$
$x = 16$	

Example 12.

$7x - 11 = 45$	$+ 11$
$7x = 56$	$\div 7$
$x = 8$	

8. Solve the equations.

a. $2x + 5 = 27$ $=$	**b.** $3x - 8 = 34$ $=$	**c.** $7x + 5 = 54$ $=$
d. $10z - 7 = 97$ $=$	**e.** $832 = 3x + 85$ $=$	**f.** $56 + 21 = 5x - 3$ $=$