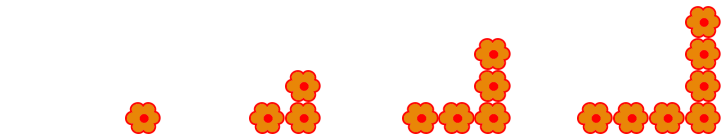
Growing Patterns



How do you think this pattern is growing?

How many flowers will there be in step 39?

This pattern adds 2 flowers in each step, except in step 1. This means that by step 39, we have added 2 flowers 38 times. Therefore, there are $1 + 2 \cdot 38 = 77$ flowers in step 39.

Write a formula for the number of flowers in step *n*.

There are several ways to do this. The three ways explained below are not the only ones!

1. Let's view the pattern as adding 2 flowers in each step after the first one. By step n, the pattern has added one less than n times 2 flowers, because we need to exclude that first step. This means that $(n - 1)$ times 2, or $(n - 1) \cdot 2$, flowers added to the one flower that we started with.

   This gives us the expression $1 + (n - 1) \cdot 2$. Since we customarily put the variable first and the constant last, we can rewrite that expression as $1 + 2(n - 1)$ and then as $2(n - 1) + 1$.

2. Another way to think about this pattern is as two legs. One leg includes the flower in the corner, so it has the same number of flowers as the step number. The other leg doesn't have the corner flower, so it has one flower less than the step numbers. In other words, in step 3, we have 3 + 2 flowers. In step 4, we have 4 + 3 flowers. In step 5, we have 5 + 4 flowers.

   This gives us a formula for the number of flowers in step n: there are $n + (n - 1)$ flowers in step n.

3. Yet another way is that, in each step, there are twice as many flowers as the step number, minus one for the flower that is shared. For example, in step 4, we have twice 4 minus 1, which is seven flowers.

   This also gives us a formula: there are $2n - 1$ flowers in step n.

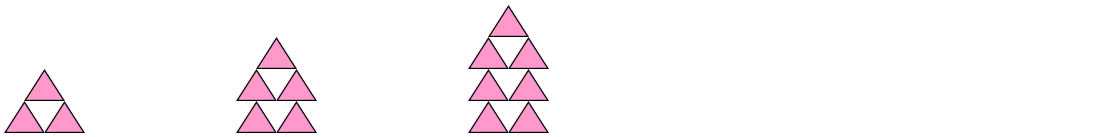
All of the formulas are equivalent (just as we would expect!) and simply represent different ways of thinking about the number of flowers in each step. On the right, you can see how the first two formulas can be simplified to the third one.

$n + (n - 1)$	$2(n - 1) + 1$
$= n + n - 1$	$= 2n - 2 + 1$
$= 2n - 1$	$= 2n - 1$

In which step are there 583 flowers?

We can use our formula to write an equation to answer this question. In the question the step number n is unknown, but the total number of flowers in that step is 583. Since we know from our formula that there are $2n - 1$ flowers in step n, we get

$$\begin{aligned} 2n - 1 &= 583 && +1 \\ 2n &= 584 && \div 2 \\ n &= 292 \end{aligned}$$



1 2 3 4 5

1. **a.** How is this pattern growing?

b. How many triangles will there be in step 39?

c. Write a formula for the number of triangles in step *n*.
Check your answer with your teacher before going on to part (d).

d. In which step will there be 311 triangles?
Write an equation and solve it.
Notice, this question is <u>different</u> from the one in part (c).

1 2 3 4 5

2. **a.** How do you think this pattern is growing?

b. How many snowflakes will there be in step 39?

c. Write a formula for the number of snowflakes in step *n*.
Check your answer with your teacher before going on to part (d).

d. In which step will there be 301 snowflakes?
Write an equation and solve it.

Sample worksheet from
www.mathmammoth.com