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Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Some of them, such as fractions and decimals, students are familiar with, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers).

The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: summarizing distributions using measures of center and variability.

This book, 6-B, covers number theory topics (chapter 6), fractions (chapter 7), integers (chapter 8), geometry (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 6-A worktext.

Chapter 6 first reviews prime factorization and then applies those principles to using the greatest common factor to simplify fractions and the least common multiple to find common denominators. Chapter 7 provides a thorough review of the fraction operations from fifth grade, and includes ample practice in solving problems with fractions.

Chapter 8 introduces students to integers. Students plot points in all four quadrants of the coordinate plane, reflect and translate simple figures, and learn to add and subtract with negative numbers. (Multiplication and division of integers will be studied in 7th grade.)

The next chapter, Geometry, focuses on calculating the area of polygons. The final chapter is about statistics. Beginning with the concept of a statistical distribution, students learn about measures of center and measures of variability. They also learn how to make dot plots, histograms, and boxplots, as ways to summarize and analyze distributions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2	10
6-B	157	88	10	78	2	10
Grade 6 total	323	180	20	160	2	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional review before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional review. Then, to get a count of "pages/day", divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let's assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems ("drill") or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Prime Factorization, GCF and LCM
Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we review prime factorization and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over into algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as 27 + 45. For example, a sum like 27 + 45 factors into 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is in finding the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures," and you can let students read more about them by accessing the Internet resources mentioned below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list for Internet resources.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

You can find some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 6

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch6

SCAN ME

The Sieve of Eratosthenes and Prime Factorization

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in 4th and 5th grades... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________	**b.** 60 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________
c. 84 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________	**d.** 97 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________

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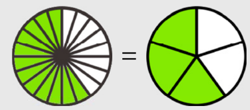
Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the “same amount of pie” as it was before. It is just cut differently.



Every four slices have been joined together.

$$\overset{\div 4}{\underset{\div 4}{\frac{12}{20} = \frac{3}{5}}}$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →

Notice that here, the 4’s that were cancelled out do ***not*** get indicated in any way! You only think it: “I divide 12 by 4, and get 3. I divide 20 by 4, and get 5.”

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5’s, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Leah simplified various fractions like you see below. She did not get them right though. Explain to her what she is doing wrong.

$$\frac{24}{84} = \frac{20}{80} = \frac{1}{4} \qquad \frac{27}{60} = \frac{7}{40} \qquad \frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$$

Using factoring when simplifying

Carefully study the example on the right where we simplify the fraction 144/96 to **lowest terms --** in other words, where the numerator and the denominator have no common factors.

- First we factor (write) 144 as 12 · 12 and 96 as 8 · 12.
- Then we simplify in two steps:
    1. 12 and 8 are both divisible by 4, so they simplify into 3 and 2.
    2. 12 and 12 are divisible by 12, so they simplify into 1 and 1. Essentially, they cancel each other out.
- Lastly we write the improper fraction 3/2 as a mixed number.

$$\frac{144}{96} = \frac{\overset{3}{\cancel{12}} \cdot \overset{1}{\cancel{12}}}{\underset{2}{\cancel{8}} \cdot \underset{1}{\cancel{12}}} = \frac{3}{2} = 1\frac{1}{2}$$

For a comparison, here is another way to write the simplification in several steps, and that you've seen in earlier grades in Math Mammoth:

$$\frac{144}{96} \underset{\div 12}{\overset{\div 12}{=}} \frac{12}{8} \underset{\div 4}{\overset{\div 4}{=}} \frac{3}{2} = 1\frac{1}{2}$$

Let's study some more examples. (Remember that they don't show the number that you divide by.)

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{5}{\cancel{35}} \cdot \underset{1}{\cancel{3}}} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\overset{3}{\cancel{9}} \cdot \overset{1}{\cancel{5}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

3. Simplify. Write the simplified numerator above and the simplified denominator below the old ones.

a. $\frac{14}{16}$	**b.** $\frac{33}{27}$	**c.** $\frac{12}{26}$	**d.** $\frac{9}{33}$	**e.** $\frac{42}{28}$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify. Also, give your final answer as a mixed number, if applicable.

a. $\frac{56}{84} = \frac{7 \cdot 8}{21 \cdot 4} =$	**b.** $\frac{54}{144} = \frac{6 \cdot 9}{12 \cdot 12} =$	**c.** $\frac{120}{72} = \frac{10 \cdot \boxed{}}{\boxed{} \cdot 9} =$
d. $\frac{80}{48} = \frac{\boxed{} \cdot 8}{\boxed{} \cdot 8} =$	**e.** $\frac{36}{90} = \frac{\qquad}{\qquad} =$	**f.** $\frac{28}{140} = \frac{\qquad}{\qquad} =$

5. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	**b.** $\frac{66}{82}$	**c.** $\frac{69}{99}$
d. $\frac{120}{600}$	**e.** $\frac{38}{52}$	**f.** $\frac{72}{84}$

Simplify "criss-cross"

These examples are from the previous page. This time the 45 in the numerator has been written as 5 · 9 instead of 9 · 5. We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

This same concept can be applied to make multiplying fractions easier.

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \cdot \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \cdot \underset{5}{\cancel{35}}} = \frac{2}{5}$$

6. Simplify. Give your final answer as a mixed number, if applicable.

a. $\frac{14}{84} = \frac{2 \cdot 7}{21 \cdot 4} =$	**b.** $\frac{54}{150} = \frac{9 \cdot \boxed{}}{10 \cdot \boxed{}} =$	**c.** $\frac{138}{36} = \frac{2 \cdot \boxed{}}{\boxed{} \cdot 4} =$
d. $\frac{27}{20} \cdot \frac{10}{21} =$	**e.** $\frac{75}{90} = \underline{} =$	**f.** $\frac{48}{45} \cdot \frac{55}{64} =$

Example 3. Here, the simplification is done in two steps.
In the first step, 12 and 2 are divided by 2, leaving 6 and 1.
In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot 69} = \frac{\overset{2}{\cancel{6}} \cdot 4}{1 \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

$$\frac{48}{138} = \frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

7. Simplify the fractions to lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	**b.** $\frac{84}{102}$	**c.** $\frac{85}{105}$
d. $\frac{8}{5} \cdot \frac{8}{20} =$	**e.** $\frac{72}{120}$	**f.** $\frac{104}{240}$
g. $\frac{35}{98}$	**h.** $\frac{5}{7} \cdot \frac{17}{15} =$	**i.** $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

Chapter 7: Fractions
Introduction

This chapter begins with a review of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, “How many times does this number fit into that number?” For example, we can write a division from the question, “How many times does 1/3 fit into 1?” The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \cdot (1/3) = 1$. These two numbers, 3/1 and 1/3, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. Thinking of fitting the divisor into the dividend (measurement division) also gives us a tool to check whether the answer to a division problem is reasonable.

Naturally, the lessons also present the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the “invert (flip)-and-multiply” rule. However, that “rule” is just a shortcut. It is necessary to memorize it, but memorizing a shortcut doesn’t help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are review in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

Also, don’t forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems
I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 7

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch7

SCAN ME

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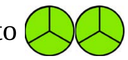
Dividing Fractions: Reciprocal Numbers

One interpretation of division is **measurement division**, where we think: *How many times does one number go into another?* For example, to solve how many times 11 fits into 189, we divide 187 ÷ 11 = 17.

(The other interpretation is equal sharing; we will come to that later.)

Let's apply that to fractions. How many times does  go into  ?

We can solve this just by looking at the pictures: three times. We can write the division: $2 \div \frac{2}{3} = 3$.

To check the division, we multiply: $3 \cdot \frac{2}{3} = \frac{6}{3} = 2$. Since we got the original dividend, it checks.

We can use measurement division to check whether an answer to a division is reasonable.

For example, if I told you that $7 \div 1\frac{2}{3}$ equals $14\frac{1}{3}$, you can immediately see it doesn't make sense:

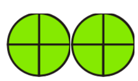
1 2/3 surely does not fit into 7 that many times. Maybe three to four times, but not 14!

You could also multiply to see that: 14-*and-something* times 1-*and-something* is way more than 14, and closer to 28 than to 14, instead of 7.

1. Find the answers that are unreasonable without actually dividing.

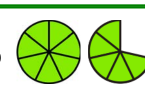
a. $\frac{4}{5} \div 6 = \frac{2}{15}$	**b.** $2\frac{3}{4} \div \frac{1}{4} = \frac{7}{12}$	**c.** $\frac{7}{9} \div 2 = \frac{7}{18}$	**d.** $8 \div 2\frac{1}{3} = 18\frac{1}{3}$	**e.** $5\frac{1}{4} \div 6\frac{1}{2} = 3\frac{1}{8}$

2. Solve with the help of the visual model, checking how many times the given fraction fits into the other number. Then write a division. Lastly, write a multiplication that checks your division.

a. How many times does  go into  ?

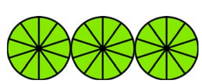
$2 \div \frac{3}{4} =$

Check: ____ $\cdot \frac{3}{4} =$

b. How many times does  go into  ?

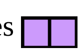
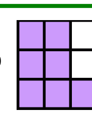
___ ___ ÷ ___ =

Check:

c. How many times does  go into  ?

3 ÷ ___ =

Check:

d. How many times does  go into  ?

___ ÷ ___ =

Check:

3. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$5 \div \frac{1}{4}$ $\downarrow \quad \downarrow$ $5 \cdot 4 = 20$	$3 \div \frac{1}{8}$ $\downarrow \quad \downarrow$ $3 \cdot 8 = 24$	$9 \div \frac{1}{7}$ $\downarrow \quad \downarrow$ $9 \cdot 7 = 63$

Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem 5 ÷ (1/4).
Since 1/4 goes into 1 exactly four times, it must go into 5 exactly 5 · 4 = 20 times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \cdot 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \cdot \frac{n}{m} = \frac{n \cdot m}{m \cdot n} = \frac{m \cdot n}{m \cdot n} = 1$.

To find the reciprocal of a mixed number or a whole number, first write it as a fraction, then “flip” it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$. And since $28 = \frac{28}{1}$, its reciprocal number is $\frac{1}{28}$.

4. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

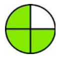
a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\frac{7}{8}$	**d.** 32	**e.** $2\frac{1}{8}$
$\frac{5}{8} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$32 \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$

5. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**c.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**d.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$	**e.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does  go into  ?

$$\frac{3}{4} \div \frac{1}{3}$$

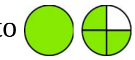
$$\downarrow \; \downarrow$$

$$\frac{3}{4} \cdot 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes,  fits into  a little more than two times.

How many times does  go into  ?

$$\frac{7}{4} \div \frac{2}{5}$$

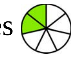
$$\downarrow \; \downarrow$$

$$\frac{7}{4} \cdot \frac{5}{2} = \frac{35}{8} = 4\;\frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes.  goes into 1 3/4 over four times.

How many times does  go into  ?

$$\frac{2}{9} \div \frac{2}{7} =$$

$$\downarrow \; \downarrow$$

$$\frac{\cancel{2}}{9} \cdot \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because  does not go into  even one full time!

Remember: There are ***two*** **changes** in each calculation:

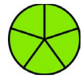
1. **Change the division into multiplication**.
2. **Use the reciprocal of the divisor**.

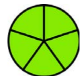
6. Solve these division problems using the shortcut. Remember to check to make sure your answer makes sense.

a. $\frac{3}{4} \div 5$ $\downarrow \; \downarrow$ $\frac{3}{4} \cdot \frac{1}{5} =$	**b.** $\frac{2}{3} \div \frac{6}{7}$
c. $\frac{4}{7} \div \frac{3}{7}$	**d.** $\frac{2}{3} \div \frac{3}{5}$
e. $4 \div \frac{2}{5}$	**f.** $\frac{13}{3} \div \frac{1}{5}$

Now let’s try to **make some sense <u>visually</u>** out of how reciprocal numbers fit into the division of fractions.

Example 1. We can think of the division **1 ÷ (2/5)** as asking, **“How many times does 2/5 fit into 1?”**

Using pictures: How many times does  go into  ? (From the looks of it, at least two times!)

From the picture we can see that  goes into  two times, and then we have 1/5 left over.

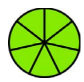
But how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

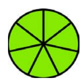
That is like trying to fit a TWO-part piece into a hole that holds just ONE part.
Only 1/2 of the two-part piece fits! So, 2/5 fits into 1/5 exactly half a time.

So we found that, in total, 2/5 fits into 1 exactly **2 ½ times**. We can write the division $1 \div \frac{2}{5} = 2\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \cdot \frac{2}{5} = 1$. Reciprocals!

Example 2. We can think of the division **1 ÷ (5/7)** as, **“How many times does 5/7 fit into 1?”**

Using pictures: How many times does  go into  ? (It looks like, a bit over one time.)

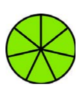
From the picture we can see that  goes into  just once, and then we have 2/7 left over.

But how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The five-part piece fits into a hole that is only big enough for two parts just 2/5 of the way.

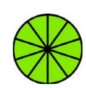
So 5/7 fits into one exactly 1 2/5 times—and this makes sense because, as we noted at first, it looked like 5/7 fit into one a little over one time. The division is $1 \div \frac{5}{7} = 1\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$. Reciprocals again!

7. Write a division.

a. How many times does  go into  ?

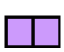
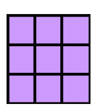
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

b. How many times does  go into  ?

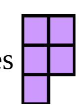
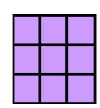
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

c. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

d. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

8. Fill in the answers and complete the patterns. You will be able to do a lot of these in your head!

a.	**b.**	**c.**	**d.**
$3 \div \frac{1}{5} =$	$6 \div \frac{1}{4} =$	$1 \div \frac{1}{4} =$	$8 \div \frac{1}{2} =$
$3 \div \frac{2}{5} =$	$6 \div \frac{2}{4} =$	$2 \div \frac{1}{4} =$	$8 \div \frac{2}{2} =$
$3 \div \frac{3}{5} =$	$6 \div \frac{3}{4} =$	$3 \div \frac{1}{4} =$	$8 \div \frac{3}{2} =$
$3 \div \frac{4}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$
$3 \div \frac{5}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$

Epilogue (optional)

The lesson didn't go into full details as to why multiplication by the reciprocal always gives us the answer to a division problem. Let's continue that discussion a bit.

Any division can be turned into a multiplication. For example, from the division $2 \div \frac{3}{4} =$ ______ , we can write the multiplication $\frac{3}{4} \cdot$ ______ $= 2$.

To find what goes on the empty line, we can first of all put there the reciprocal of ¾ : $\frac{3}{4} \times \underline{\frac{4}{3} \times \quad\quad} = 2$.

Notice the multiplication of the two fractions above equals 1 (since they are reciprocals). To make the left side of the equation equal 2, we place 2 in the empty line: $\frac{3}{4} \times \underline{\frac{4}{3} \times \ 2\ } = 2$

And thus, the answer to the original division is $\frac{4}{3} \cdot 2$ (or $2 \cdot \frac{4}{3}$) — which is the original dividend times the reciprocal of the divisor.

Let's take another example: $2\frac{1}{6} \div 5\frac{3}{4} =$ ______

We turn it around and make a multiplication problem: $5\frac{3}{4} \cdot$ ______ $= 2\frac{1}{6}$.

Now, 5 ¾ = 23/4. So first, we insert the reciprocal of 23/4, which is 4/23: $5\frac{3}{4} \cdot \frac{4}{23} \cdot$ ______ $= 2\frac{1}{6}$.

Since 5 ¾ · (4/23) = 1, then the number we still need to put on the empty line must be 2 1/6, and thus the answer to the original division problem is 4/23 · (2 1/6), or (2 1/6) · 4/23 — the original divided times the reciprocal of the divisor.

We could use a similar argument to show in the general case that the answer to any division problem $a \div b$ is always going to be a times the reciprocal of b.

Divide Fractions

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

This shortcut works *in every case*, whether the numbers involved are whole numbers, fractions, or mixed numbers.

$\frac{2}{5} \div \frac{7}{9}$ $\downarrow \ \downarrow$ $\frac{2}{5} \cdot \frac{9}{7} = \frac{18}{35}$ Check: $\frac{18}{35} \cdot \frac{7}{9} = \frac{2}{5}$	$7 \div \frac{9}{10}$ $\downarrow \ \downarrow$ $7 \cdot \frac{10}{9} = \frac{70}{9} = 7\frac{7}{9}$ Check: $\frac{70}{9} \cdot \frac{9}{10} = \frac{7}{1} = 7$	$1\frac{10}{11} \div 5$ $\downarrow \ \downarrow$ $\frac{21}{11} \cdot \frac{1}{5} = \frac{21}{55}$ Check: $\frac{21}{55} \cdot 5 = \frac{21}{11} = 1\frac{10}{11}$

Notice: when you check the problems, you will need to use the *original* divisor, not the “flipped” one.

1. Solve. Change mixed numbers to fractions before dividing. Check each division by multiplication.

a. $\frac{9}{10} \div \frac{2}{5}$ Check:	**b.** $\frac{3}{7} \div \frac{4}{3}$ Check:	**c.** $\frac{2}{11} \div \frac{2}{3}$ Check:
d. $1\frac{7}{8} \div \frac{3}{4}$ Check:	**e.** $2\frac{1}{15} \div 1\frac{3}{5}$ Check:	**f.** $5\frac{10}{11} \div 6$ Check:

2. How many 2/3 cup servings can you get out of 5 cups of ice cream?

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Problem Solving with Fractions 1

1. **a.** Anna needs to make 30 servings of spiced coffee for a party. Calculate the amount of each ingredient she needs.

> Spiced Coffee – 4 servings
>
> 1 1/2 teaspoons of ground cinnamon
> 1/2 of a teaspoon of ground nutmeg
> 2 tablespoons of sugar
> 1 cup of heavy cream
> 3 cups of coffee
> 4 teaspoons of chocolate syrup

b. Next week she wants to make just *one* serving for herself. Calculate the amount of each ingredient she needs.

2. Sam planted tomatoes in his garden, which is a rectangle with an area of 2 ½ m^2. If one side of the garden measures 5 m, how long is the other side?

3. Which is a better deal: a book that costs $45.55 at 1/5 off or a book that costs $52.80 at 1/4 off?

Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants and integer addition and subtraction. The multiplication and division of integers will be studied next year.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules", for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number-line and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 8

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

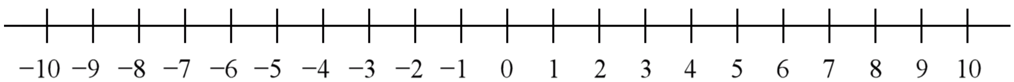
We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch8

SCAN ME

Integers

When we continue the number-line towards the left from zero, we come to the **negative numbers**.



The **negative whole numbers** are −1, −2, −3, −4 and so on.
The **positive whole numbers** are 1, 2, 3, 4 and so on. You can also write them as +1, +2, +3, *etc*.
Zero is neither positive nor negative.
All of the negative and positive whole numbers and zero are called **integers**.

Read −1 as "negative one" and −5 as "negative five". Some people read −5 as "minus five".
That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

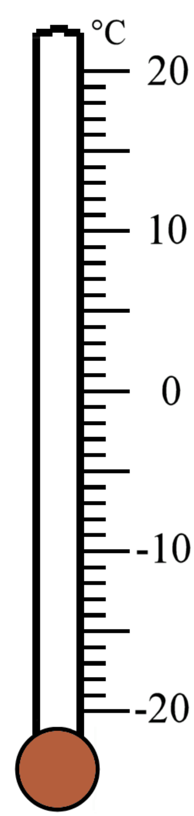
Put a "−" sign in front of negative numbers. This sign can also be elevated: $^{-}5$ is the same as −5.

Often, we need to put brackets around negative numbers in order to avoid confusion with other symbols. Therefore, $^{-}5$, −5 and (−5) all mean "negative five".

Negative numbers are commonly used with temperature. They are also used to express debt. If you owe \$5, you write that as −\$5. Another use is with elevation below sea level. For example, just as 200 m can mean an elevation of 200 meters above sea level, −100 m would mean 100 meters *below* sea level.

1. Plot the integers on the number-line.

   a. −7 **b.** +6 **c.** −4 **d.** −2



2. Write an integer appropriate to each situation.

   a. Daniel owes \$23.

   b. Mary earned \$250.

   c. The airplane flew at the altitude of 8 800 meters.

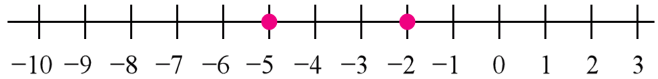
   d. The temperature in the freezer is 18 degrees Celsius below zero.

   e. A dolphin dove 9 m below sea level.

3. The temperature changed from what it was before. Find the new temperature.
   You can draw the mercury on the thermometer to help you.

before	1°C	2°C	−2°C	−4°C	−12°C	−8°C
change	drops 3°C	drops 7°C	drops 1°C	rises 5°C	rises 4°C	rises 3°C
now						

Which is more, −5 or −2?

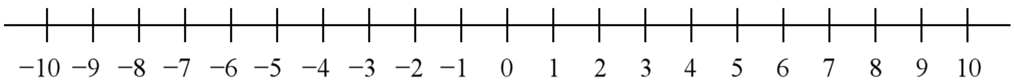


Which is *warmer*, −5°C or −2°C? Clearly −2°C is. Temperatures just get colder and colder the more you move towards the negative numbers. We can write a comparison: −2°C > −5°C.

Which is the *better* money situation, to have −$5 (owe $5) or to have −$2 (owe $2)? Clearly, it is better to owe only $2 because you can pay that off easier. We can write: −$5 < −$2.

Which is the *higher* elevation, −5 m or −2 m? Of course, 2 m below sea level, or −2m, is higher.

On the number line, the number that is ***farther to the right*** is the **greater** number. So, −5 < −2.



4. Compare. Write **<** or **>** between the numbers. You can plot the integers on the number line to help you.

a. −2 ☐ −3	**b.** 8 ☐ −8	**c.** −3 ☐ 0	**d.** 4 ☐ −3	**e.** −5 ☐ −9
f. −10 ☐ −30	**g.** −4 ☐ 1	**h.** 0 ☐ −13	**i.** −2 ☐ −7	**j.** −11 ☐ −14

5. You can use the number line to help you. Which integer is ...

a. 2 more than −4 **b.** 5 more than −3 **c.** 3 less than 1 **d.** 6 more than −11

6. Find the number that is 5 less than ... **a.** 0 **b.** −3 **c.** 3

7. Express the situations using integers. Then write > or < to compare them.

a. Shelly owes $10 and Mary owes $8.

b. One fish was swimming 3 m below the surface of the water, and another fish was swimming 4 m below the surface of the water.

c. The temperature this morning was 10°C below zero. Now it is 6°C below zero.

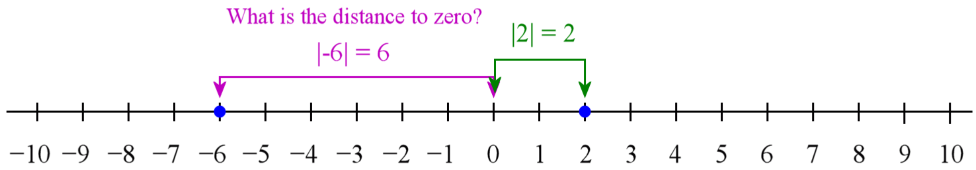
d. Henry has $5. Emma owes $5.

e. The temperature during the day was 10°C but at night it was 2°C below zero.

8. Write the numbers in order from the least to the greatest.

a. −2 0 −4 4	**b.** −3 −6 5 3
c. −20 −10 −14 −9	**d.** −3 0 −6 −8

The **absolute value** of a number is its distance from zero.



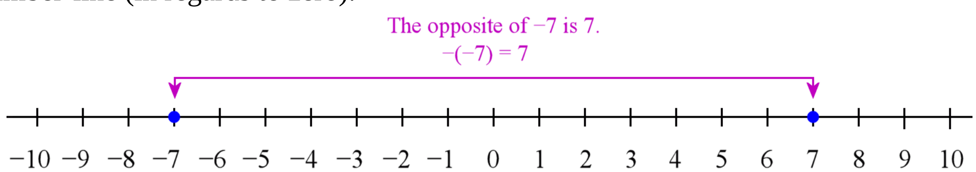
We denote the absolute value of a number using vertical bars around the number.

So, $|-4|$ means "the absolute value of 4", which is 4. Similarly, $|87| = 87$.

9. Find the absolute values of these numbers.

a. $|-5|$ **b.** $|-12|$ **c.** $|7|$ **d.** $|0|$ **e.** $|68|$

The **opposite** of a number is the number that is at the same distance from zero as it is, but on the *opposite* side of the number-line (in regards to zero).



We denote the opposite of a number using the minus sign. For example, −4 means the opposite of 4, which is −4. Or, −(−2) means the opposite of negative two, which is 2.

The opposite of zero is zero itself. In symbols, −0 = 0.

"But wait," you might ask, "doesn't −4 mean negative four, not the 'opposite of four'?"

It can mean either. Sometimes the context will help you to differentiate between the two (to tell which is which). Other times it's unnecessary to differentiate because, after all, the opposite of four is negative four: −4 = −4. ☺

So there are three different meanings for the minus sign:

1. To indicate subtraction: 7 − 2 = 5.
2. To indicate negative numbers: "negative 7" is written −7.
3. To indicate the opposite of a number: −(−14) is the opposite of negative 14.

10. Think of the minus sign as signifying "the opposite of". Simplify.

a. −5 **b.** −(−9) **c.** −10 **d.** −0 **e.** −(−100)

11. Write using mathematical symbols, and simplify (solve) if possible.

a. the opposite of 6

b. the opposite of the absolute value of 6

c. the absolute value of negative 6

d. the absolute value of the opposite of 6

e. the opposite of −6.

f. the absolute value of the opposite of −6

Coordinate Grid

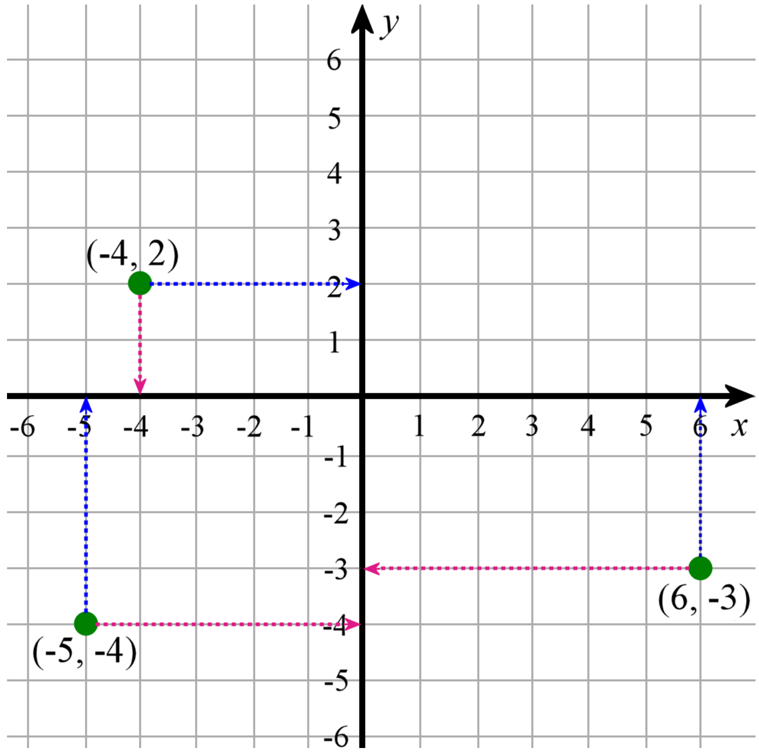
This is the *coordinate grid* or *coordinate plane*. We have extended the *x*-axis and the *y*-axis to include negative numbers now. The axes cross each other at the *origin*, or the point (0, 0).

The axes divide the coordinate plane into four parts, called *quadrants*. Previously you have worked in only the so-called first quadrant, but now we will use all four quadrants.

The coordinates of a point are found in the same manner as before. Draw a vertical line (either up or down) from the point towards the *x*-axis. Where this line crosses the *x*-axis tells you the point's *x*-coordinate.

Similarly, draw a horizontal line (either right or left) from the point towards the *y*-axis. Where this line crosses the y-axis tells you the point's *y*-coordinate.

We list first the point's *x*-coordinate and then the *y*-coordinate. Look at the examples in the picture.



1. Write the *x*- and *y*-coordinates of the points.

A (____ , ____)

B (____ , ____)

C (____ , ____)

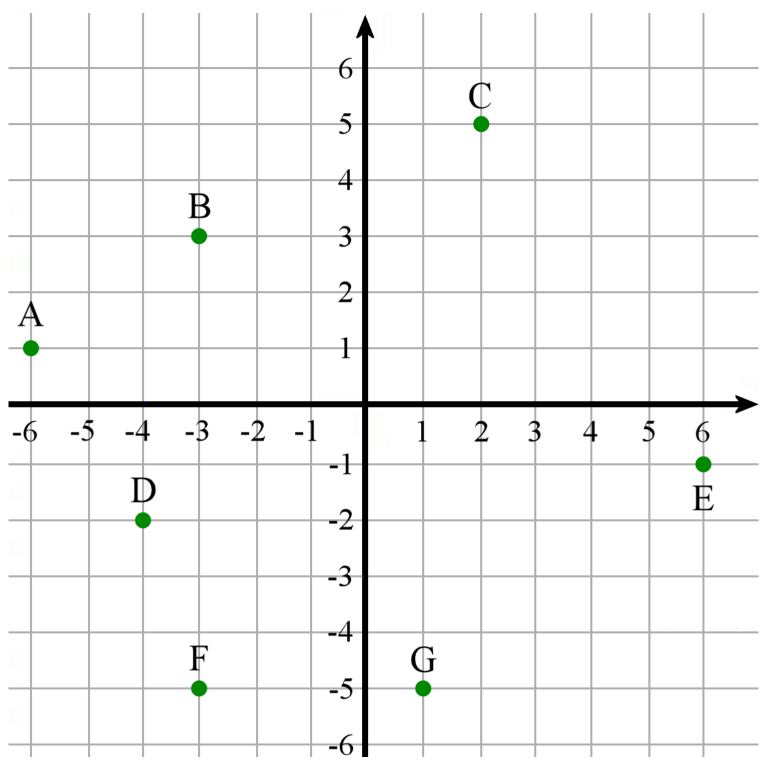
D (____ , ____)

E (____ , ____)

F (____ , ____)

G (____ , ____)

Self-check: Add the *x*-coordinates of all points. You should get −7.



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Graphing

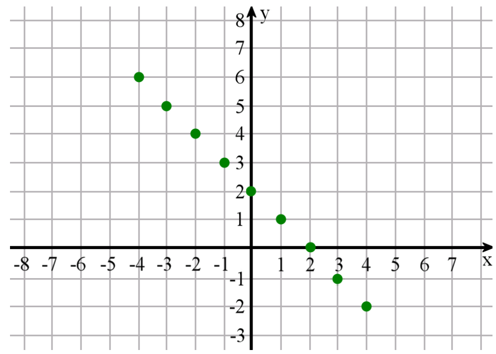
Remember? When an equation has two variables, there are many values of x and y that make that equation true.

Example. Note the equation $y = 2 - x$. If $x = 0$, then we can calculate the value of y using the equation: $y = 2 - 0 = 2$.

So, when $x = 0$ and $y = 2$, that equation is true. We can plot the number pair (0, 2) on the coordinate grid.

Some of the other (x, y) values that make the equation true are listed below, and they are plotted on the right.

x	−4	−3	−2	−1	0	1	2	3	4
y	6	5	4	3	2	1	0	−1	−2

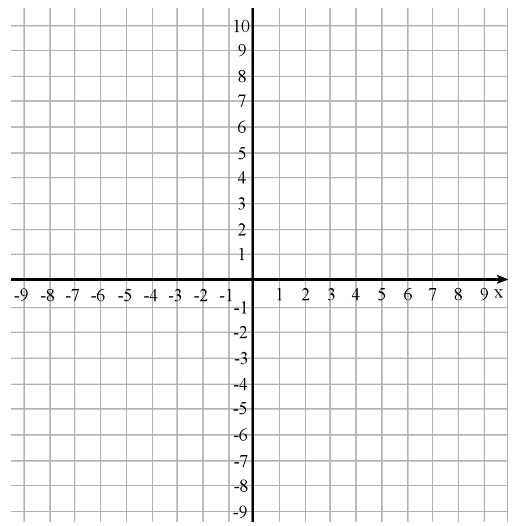


1. Plot the points from the equations. Graph both (b) and (c) in the same grid.

a. $y = x + 4$

x	−9	−8	−7	−6	−5	−4	−3	−2
y								

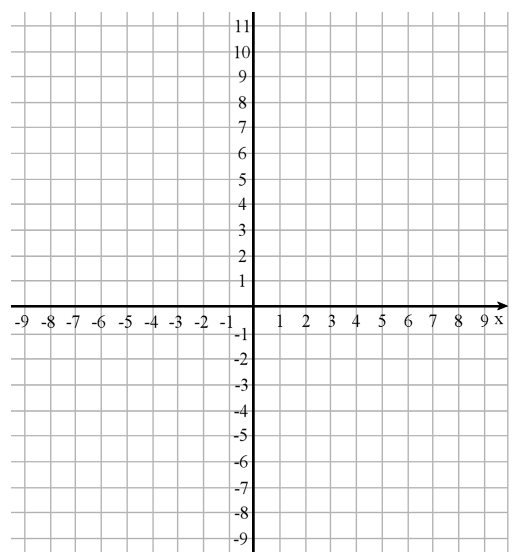
x	−1	0	1	2	3	4	5	6
y								



b. $y = 6 - x$

x	−3	−2	−1	0	1	2	3
y							

x	4	5	6	7	8	9
y						



c. $y = x - 2$

x	−5	−4	−3	−2	−1	0	1	2
y								

x	3	4	5	6	7	8	9
y							

Chapter 9: Geometry
Introduction

The main topics in this chapter include:

- area of triangles
- area of polygons
- nets and surface area of prisms and pyramids
- volume of rectangular prisms with sides of fractional length

However, the chapter starts out with some review of topics from earlier grades, as we review the different types of quadrilaterals and triangles and students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalize finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those and summing them. Students also practice their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges ½ or ⅓ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

Also, don't forget to use the resources for challenging problems: **https://l.mathmammoth.com/challengingproblems**

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 9

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch9

SCAN ME

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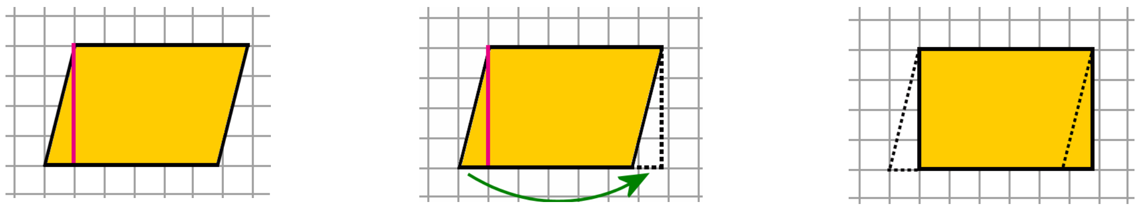
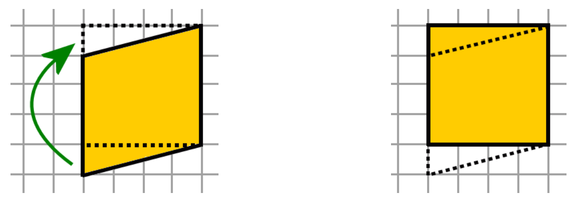
Area of Parallelograms



We draw a line from one vertex of the parallelogram in order to form a *right triangle*. Then we move the triangle to the other side, as shown. Look! We get a *rectangle*!

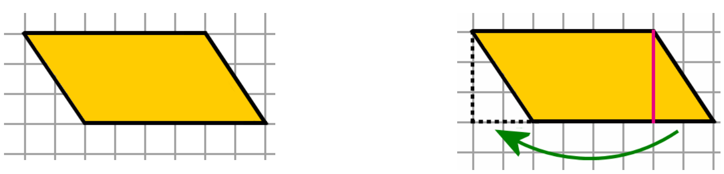
The rectangle's area is $6 \cdot 4 = 24$ square units, and that is *also the area of the original parallelogram*.

It works here, as well. The area of the rectangle and of the parallelogram are the same: both have the area of $4 \cdot 4 = 16$ square units.

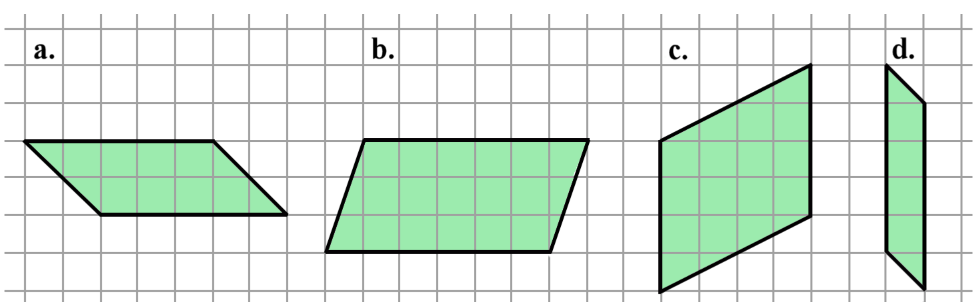


The area of a parallelogram is the same as the area of the corresponding rectangle.
You construct the rectangle by moving a right triangle from one side of the parallelogram to the other.

1. Imagine moving the marked triangle to the other side as shown. What is the area of the original parallelogram?



2. Draw a line in each parallelogram to form a right triangle. Imagine moving that triangle to the other side so that you get a rectangle, like in the examples above. Find the area of the rectangle, thereby finding the area of the original parallelogram.



a. __________ square units **b.** __________ square units

c. __________ square units **d.** __________ square units

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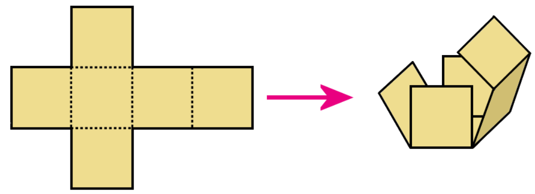
Nets and Surface Area 1

This picture shows a flat figure, called a **net**, that can be folded up to form a solid, in this case a cube.

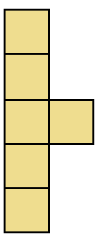
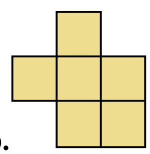
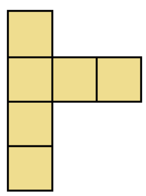
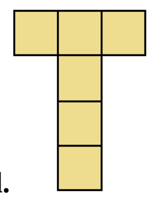
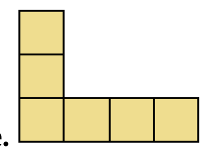
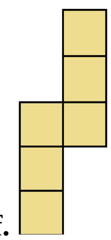


Each face of a cube is a square. If we find the total area of its faces, we will have found the **surface area** of the cube.

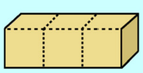
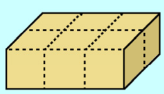
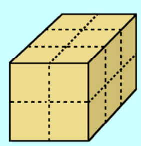
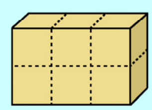
Let's say that each edge of this cube measures 2 cm. Then one face would have an area of 2 cm · 2 cm = 4 cm^2, and the total surface area of the six faces of the cube would be 6 · 4 cm^2 = 24 cm^2.

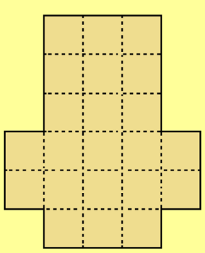
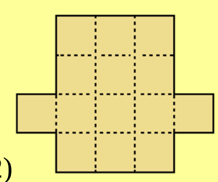
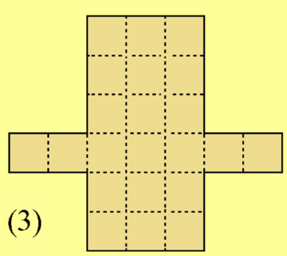
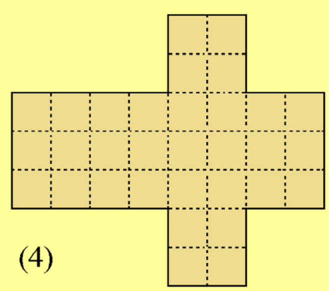
What is its volume? Remember, **volume** has to do with how much space a figure takes up, and not with "flat" area. Volume is measured in *cubic* units, whereas area is measured in *square* units. The volume of this cube is 2 cm · 2 cm · 2 cm = $(2\text{ cm})^3$ = 8 cm^3.

1. Which of these patterns are nets of a cube? In other words, which ones can be folded into a cube? You can copy the patterns on paper, cut them out and fold them.

a. 
b. 
c. 
d. 
e. 
f. 

2. Match each rectangular prism (a), (b), (c) and (d) with the correct net (1), (2), (3) and (4). Again, if you would like, you can copy the nets onto paper, cut them out, and fold them.

a. 
b. 
c. 
d. 

(1) 
(2) 
(3) 
(4) 

3. Find the surface area (A) and volume (V) of each rectangular prism in problem #2 if the edges of the little cubes are 1 cm long.

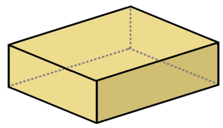
a. A = ________ cm^2 **b.** A =________ cm^2 **c.** A =________ cm^2 **d.** A =________ cm^2

V = ________ cm^3 V = ________ cm^3 V = ________ cm^3 V = ________ cm^3

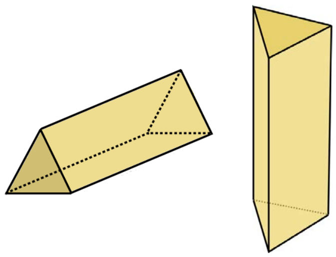
A **prism** has two identical polygons as its top and bottom faces. These polygons are called the *bases* of the prism. The bases are connected with faces that are parallelograms (and often rectangles).

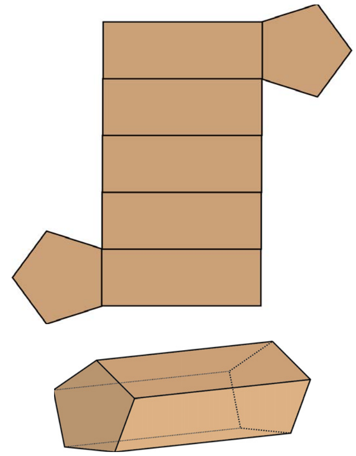
Prisms are named after the polygon used as the bases.

A rectangular prism.
The bases are rectangles.



Two **triangular prisms**.
One is lying down, where the base is facing you.
The other is “standing up”.

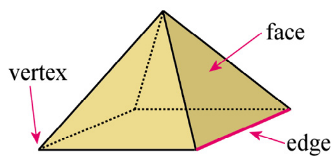


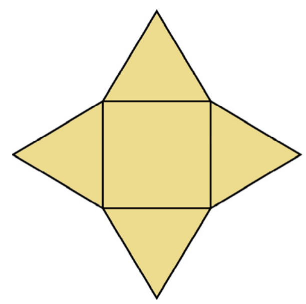


A pentagonal prism and its net. The bases are pentagons. Again, the base is not “on the bottom” but facing you.

A **pyramid** has a polygon as its bottom face (the base), and triangles as other faces.

Pyramids are named after the polygon at the base: a triangular pyramid, square pyramid, rectangular pyramid, pentagonal pyramid, and so on.



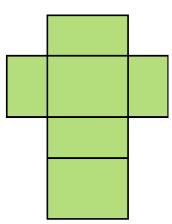


A **square pyramid** and its net.

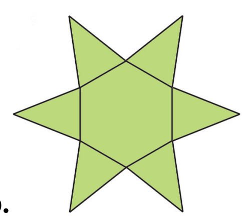
See interactive solids and their nets at the link below:
https://www.mathsisfun.com/geometry/polyhedron-models.html

4. Name the solid that can be built from each net.

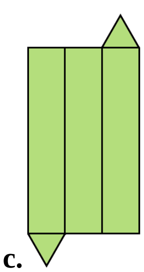
a.



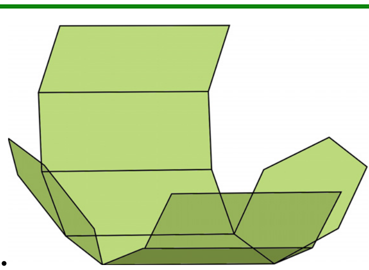
b.



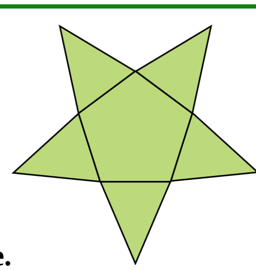
c.



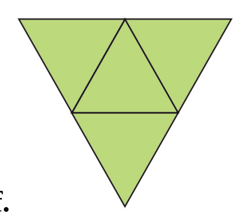
d.



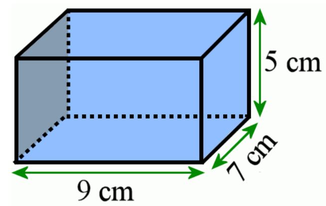
e.



f.



5. Which expression, (1), (2), or (3), can be used to calculate the surface area of this prism correctly? (You do *not* have to actually calculate the surface area.)

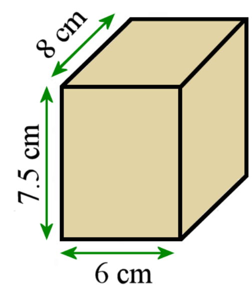


1. $2 \cdot 35\ \text{cm}^2 \ + \ 2 \cdot 63\ \text{cm}^2 \ + \ 2 \cdot 45\ \text{cm}^2$

2. $5\ \text{cm} \cdot 9\ \text{cm} \cdot 7\ \text{cm}$

3. $5\ \text{cm} \cdot 7\ \text{cm} + 9\ \text{cm} \cdot 7\ \text{cm} + 9\ \text{cm} \cdot 5\ \text{cm}$

6. Ryan organized the calculation of the surface area of this prism into three parts. Write down the intermediate calculations, and solve. This way, your teacher (or others) can follow your work. Remember also to <u>include the units</u> (cm or cm^2)!



Top and bottom:

Back and front:

The two sides:

Total:

7. The surface area of a cube is 96 square inches.

a. What is the area of one face of the cube?

b. How long is each edge of the cube?

c. Find the volume of the cube.

Puzzle Corner

Consider the rectangular prisms in problem #2. If the edges of the little cubes were double as long, how would that affect the surface area? Volume?

You can use the table below to investigate the situation.

Prism a.	Prism b.	Prism c.	Prism d.
A = _________ cm^2	A =_________ cm^2	A =_________ cm^2	A =_________ cm^2
V = _________ cm^3	V = _________ cm^3	V = _________ cm^3	V = _________ cm^3

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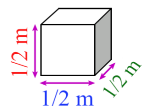
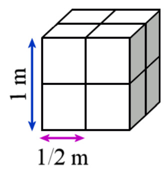
Volume of a Rectangular Prism with Sides of Fractional Length

Example 1. Let's imagine that the edges of this little cube each measure ½ m.



If we stack eight of them so that we get a bigger cube… we get this:



The bigger cube has <u>1 m edges, so its volume is 1 cubic meter</u>.

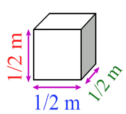
If eight identical little cubes make up this bigger cube, and its volume is 1 cubic meter, then <u>the volume of *one* little cube is 1/8 cubic meter</u>.

Notice: this is the same result that we get if we multiply the height, width and depth of the little cube:

$$\frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} = \frac{1}{8}\text{ m}^3$$

1. The edges of each little cube measure ½ m. What is the total volume, in cubic meters, of these figures?

a.



width = ___1/2___ m

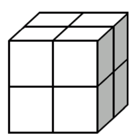
height = ________ m

depth = ________ m

___1___ little cube, 1/8 m^3

V = __________ m^3

b.



width = ________ m

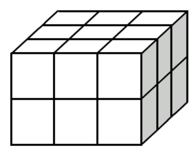
height = ________ m

depth = ________ m

___8___ little cubes, each 1/8 m^3

V = __________ m^3

c.



width = ________ m

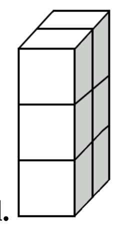
height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3

d.



width = ________ m

height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3

2. Write a multiplication (width · depth · height) to calculate the volume of the figures (c) and (d) above, and verify that you get the same result as above.

a. V = ________ m · ________ m · ________ m

=

b. V = ________ m · ________ m · ________ m

=

Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its center, spread and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarizing and analyzing distributions are its center and its variability. First we study the center, in the lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of center to the shape of the data distribution and the type of data.

Next, we study measures of variation, starting with range and interquartile range. Students use these measures in the following lesson, as they both read and draw boxplots.

The lesson *Mean Absolute Deviation* introduces students to this measure of variation. It takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program (such as Excel or LibreOffice Calc).

Next, students learn to make histograms. They will also continue summarizing distributions by describing their shape, and giving a measure of center and a measure of variability. The lesson on stem-and-leaf plots is optional.

There are some videos available for these topics at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 10

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch10

SCAN ME

Understanding Distributions

A **statistical question** is a question where we expect a range of *variability* in the answers to the question.

For example, "How old am I?" is *not* a statistical question (there is only one answer), but "How old are the students in my school?" *is* a statistical question because we expect the students' ages not to be all the same.

"How much does this TV cost?" is *not* a statistical question because we expect there to be just one answer.

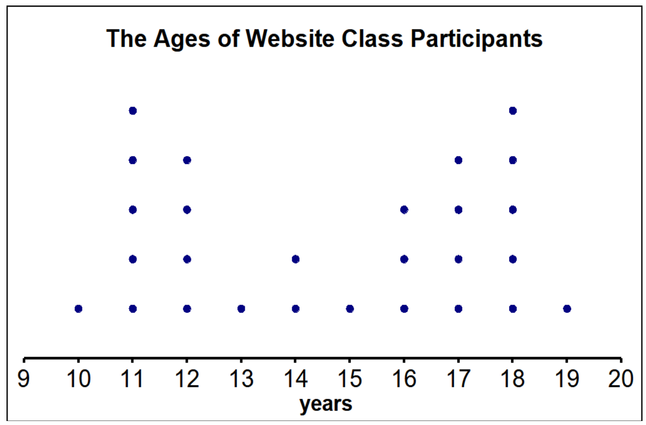
"How much does this TV cost in various stores around town?" *is* a statistical question, because we expect a number of different answers: the prices in different stores will vary.

To answer a statistical question we collect a set of **data** (many answers). The data can be displayed in some kind of a graph, such as a bar graph, a histogram, or a dot plot.

This is a **dot plot** showing the ages of the participants in a website-building class. Each dot in the plot signifies one observation. For example, we can see there was one 13-year old and two 14-year olds in the class.

The dot plot shows us the **distribution** of the data: it shows how many times (the frequency) each particular value (age in this case) occurs in the data.

This distribution is actually **bimodal,** or "double-peaked". This means it has two "centers": one around 11-12 years, and another around 17-18 years.



1. Are these statistical questions? If not, change the question so that it becomes a statistical question.

   a. What color are my teacher's eyes?

   b. How much money do the students in this university spend for lunch?

   c. How much money do working adults in Romania earn?

   d. How many children in the United States use a cell phone regularly?

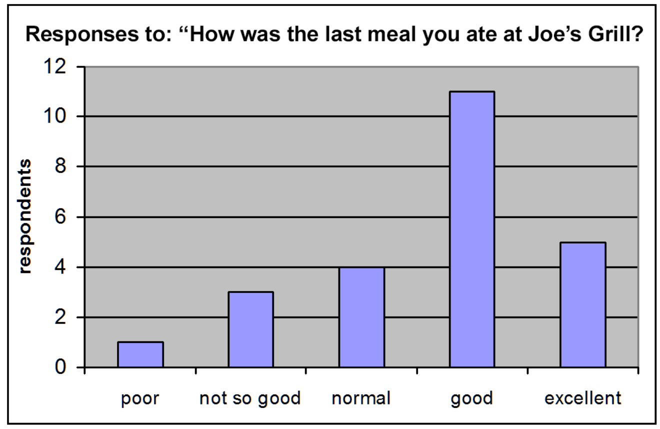
   e. What is the minimum wage in Ohio?

   f. How many sunny days were there in August, 2020, in London?

   g. How many pets does my friend have?

2. Is this graph based on a statistical question?

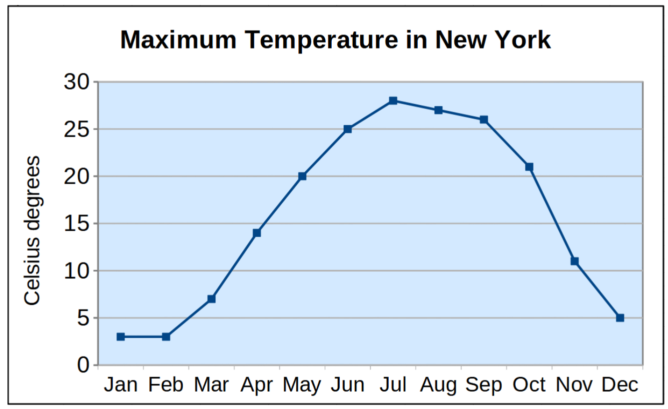
   Why or why not?



3. The line graph shows the maximum temperatures in New York for each month of a certain year.

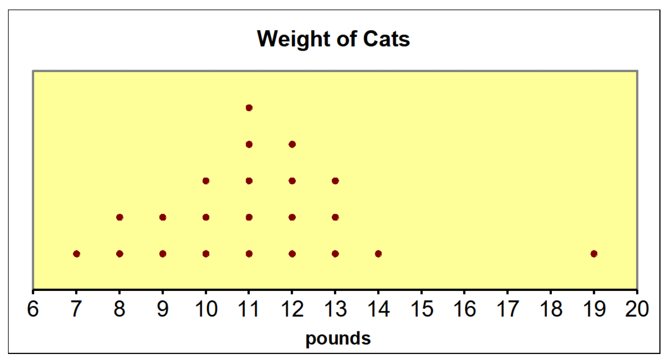
   Is this graph based on a statistical question?

   Why or why not?



4. The title of this dot plot is not the best. But, could the plot be based on a statistical question?

   If yes, give it a better, more specific, title. Imagine what situation and what question might have produced the data.



5. Change each question from a non-statistical question to a statistical question, and vice versa.

   a. What shampoo do you use?

   b. How cold was it yesterday where you live?

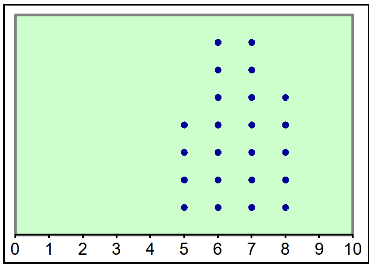
   c. How old are people in Germany when they marry (the first time)?

   d. How long does it take for our company's packages to reach the customers?

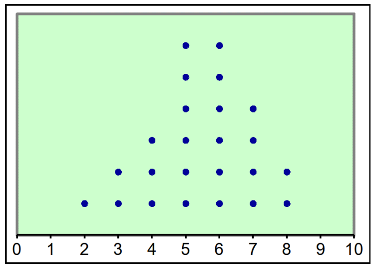
We are often interested in the **center**, **spread** and **overall shape** of the distribution. Those three things can summarize for us what is important about the distribution.

The **center** of a distribution has to do with where its peak is. We can use mean, median and mode to characterize the central tendency of a distribution. We will study those in detail in the next lesson.

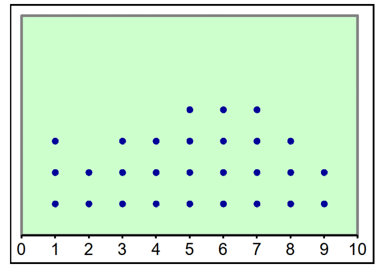
These three dot plots show how the **spread** of a distribution can vary. This means how the data items themselves are spread — whether they are "spread" all over, or tightly concentrated near some value, or somewhat concentrated around some value. We will study more about spread in another lesson.



little spread

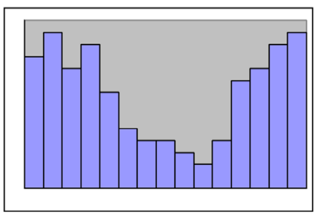


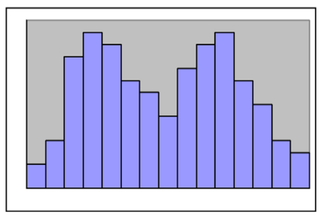
medium spread

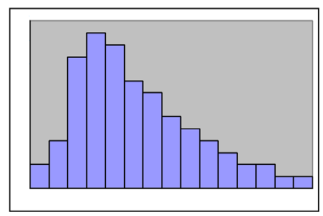


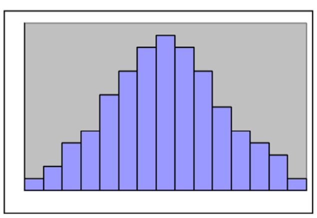
large spread

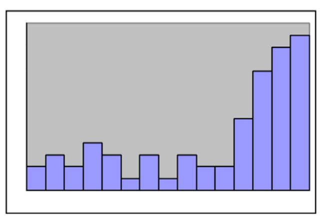
The distribution can have many varying overall **shapes.** For example:

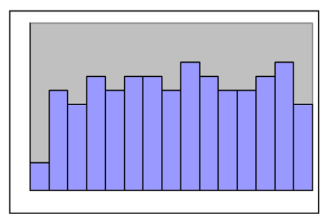

U-shaped


double-peaked (bimodal)


asymmetrical, right-tailed
(a.k.a. right-skewed)

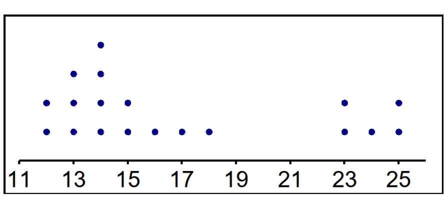

bell-shaped


J-shaped
(can also be mirrored where most of the values are at the left)

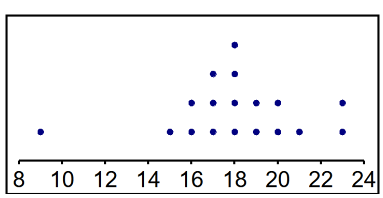

rectangular

In addition to its overall shape, a distribution may have a gap, an outlier, or a cluster:

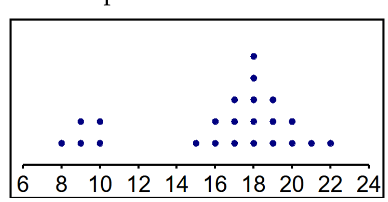
This distribution has a **gap** from 19 to 22:



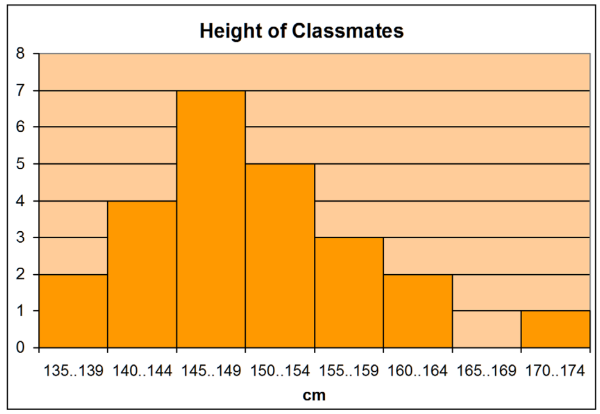
In this distribution, 9 is an **outlier —** a data item whose value is considerably less or more than all the others.



This distribution has a bell shape overall (with a peak at 18), but also a **cluster** or a smaller peak at 8-10.



6. Anne asked her classmates the question, “How tall are you?” The histogram shows the distribution of her data.



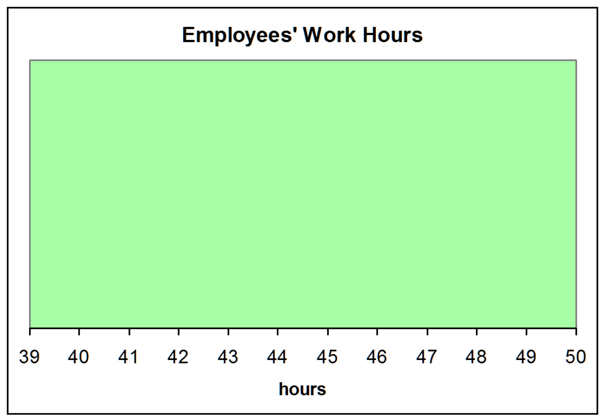
a. Describe the overall shape of the distribution, and also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

7. Make a dot plot from this data (weekly work hours of a restaurant’s employees). You need to place a dot for each observation.

48	45	46	41	42	42	43	43	42	42	41
41	45	49	40	41	41	42	46	47	42	40

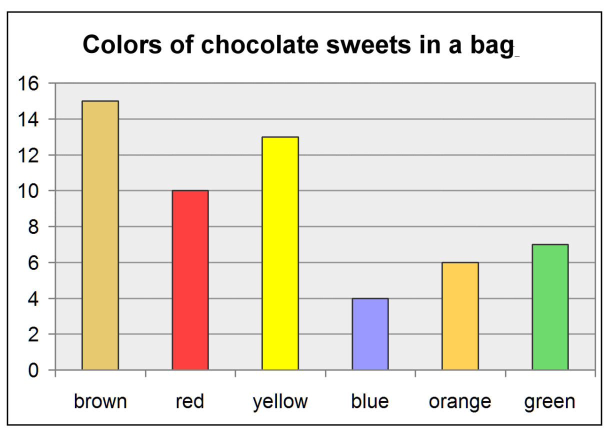


a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

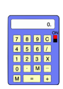
b. Where is the peak of the distribution?

c. How many observations are there?

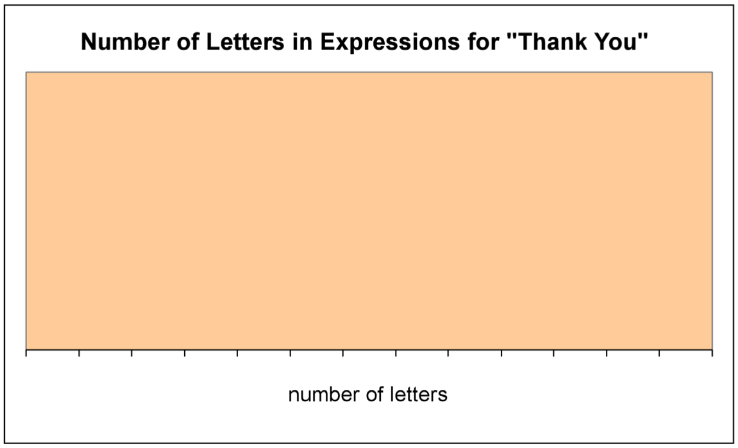
8. **a.** Does this graph show a statistical distribution? Why or why not?



b. Calculate what percentage of the candies are red and what percentage are green.



9. First, count the number of letters in these expressions for “Thank You” from various languages and fill in the empty column in the table. Next, label the number line below the dot plot so that all of the data will fit. Finally, plot the data.



Language	Spelling	Number of letters
Africaans	dankee	
Arabic	shukran	
Chinese, Cantanese	do jeh	
Chinese, Mandarin	xie xie	
Czech	dêkuji	
Danish	tak	
English	thank you	
Finnish	kiitos	
French	merci	
German	danke	
Greek	efharisto	
Hawaiian	mahalo	
Hebrew	toda	
Hindi	sukria	
Italian	grazie	
Japanese	arigato	
Korean	kamsa hamnida	
Norwegian	takk	
Philippines (Tagalog)	salamat po	
Polish	dziekuje	
Portuguese	obrigado	
Russian	spasibo	
Spanish	gracias	
Sri Lanka (Sinhak)	istutiy	
Swahili	asante	
Swedish	tack	
Thai	khop khun krab	
Turkish	tesekkür ederim	
Vietnamese	ca’m on	

a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

Mean, Median and Mode

Mean, median and **mode** are all measures for the *center* of a data set. In other words, each of them gives us a *single number* that indicates a “middle point” of the distribution.

The **mode** is the most commonly occurring data item within the data set.

- If no item occurs more often than others, there is no mode.
  Example 1. The data set {*bear, parrot, cat, dog, lizard*} has no mode.
- If two (or three, four, *etc.*) items occur equally often, there are that many modes.
  Example 2. The data set {3, 3, 6, 6, 7, 8, 8, 10} has three modes: 3, 6 and 8.

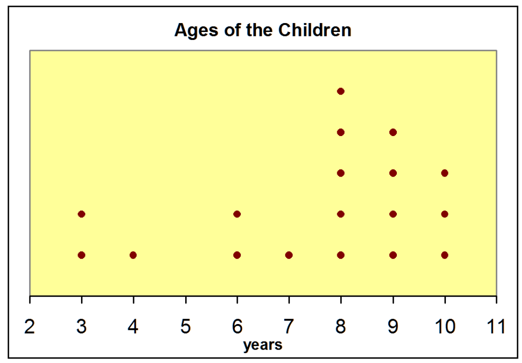
The **median** is the *middle* item after the data is organized from the least to the greatest. Exactly half of the data is before the median, and the other half is after.

- If there is an even number of data items, the median is the average of the two items in the middle.

Example 3. Find the median of the ages of a group of children:
3, 3, 4, 6, 6, 7, 8, 8, **8**, **8**, 8, 9, 9, 9, 9, 10, 10, 10

There are 18 data items and they are already in order.
The median is the “middle item”, in this case the average of the 9th and 10th ages, which are both 8. So the median is 8.
It matches well with the peak in the plot of the distribution.

What is the mode in this example?



The **mean**, or the **average**, is calculated by adding all the data items, then dividing by the number of items.

Example 4. Mia’s scores on her spelling tests were 80%, 72%, 88%, 92% and 79%.
What was her average score?

We calculate the mean by adding the five scores and dividing by 5: $\frac{80 + 72 + 88 + 92 + 79}{5} = 82.2\%$

1. Find the median and mode of these data sets.

a. 20, 25, 21, 30, 29, 24, 18, 32, 25, 26, 25 (ages of participants in a parenting class)

median ________ mode ________

b. 1, 1, 0, 2, 2, 2, 3, 1, 2, 2, 1 (the number of cars per household, for 11 households on Meadow Street)

median ________ mode ________

c. 80, 85, 80, 90, 70, 75, 90, 85, 100, 80 (Alice’s quiz scores in algebra class)

median ________ mode ________

d. sandals, crocs, tennis shoes, crocs, dress shoes, sandals (types of shoes Emma keeps on her shoe rack)

mode

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Stem-and-Leaf Plots

This lesson is optional.

A stem-and-leaf plot is made using the numbers in the data, and it looks a little bit like a histogram or a dot plot turned sideways.

In this plot, the tens digits of the individual numbers become the **stems**, and the ones digits become the **leaves**. For example, the second row 2 | 1 2 5 8 actually means 21, 22, 25, and 28. Notice how the leaves are listed in order from the smallest to the greatest.

Ages of the participants in the County Fair Karaoke Contest:

14 18 21 22 25 28 30 30
31 33 33 36 37 40 45 58

Stem	Leaf
1	4 8
2	1 2 5 8
3	0 0 1 3 3 6 7
4	0 5
5	8

4 | 5 means 45

Since stem-and-leaf plots show not only the *shape* of the distribution but also the individual *values*, they can be used to get a quick overview of the data. This distribution has a central peak and is somewhat skewed to the right.

You can also find the median fairly easily because you can follow the individual values from the smallest to the largest, and find the middle one.

Stem-and-leaf plots are most useful for numerical data sets that have 15 to 100 individual data items.

1. **a.** Complete the stem-and-leaf plot for this data:

   19 20 34 25 21 34 14 20 37 35 20 24 35 15 45 42 55

   (prices of hair dryers in three stores)

   b. What is the median?

Stem	Leaf
1	
2	
3	
4	
5	

5 | 4 means 54

2. **a.** Complete the stem-and-leaf plot for this data. This time, the stems are the first two digits of the numbers, and the leaves are the last digits.

   709 700 725 719 750 740 757 745 786 770 728 755

   (monthly rent, in dollars, for one-bedroom apartments in Houston, Texas)

   b. Find the median monthly rent.

   c. Find the interquartile range.

   d. Describe the spread of the distribution (is the data spread out a lot, a medium amount, a little, etc.)

Stem	Leaf
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	

71 | 9 means 719