Contents

Partial Products, Part 1

You can multiply the thousands, hundreds, tens, and ones separately, and then add to get the final answer.

This is called **the partial products algorithm**, or **multiplying in parts**.

7 × 326

$7 \times \underline{300} + 7 \times \underline{20} + 7 \times \underline{6}$

$2{,}100 + 140 + 42$

$= 2{,}282$

The partial products can also be written one under the other, and then added.

$$\begin{array}{rr} & 287 \\ & \times \quad 5 \\ \hline 5 \times 7 \rightarrow & 35 \\ 5 \times 80 \rightarrow & 400 \\ 5 \times 200 \rightarrow & +\ 1000 \\ \hline & 1435 \end{array}$$

1. Multiply in parts, then add.

a. 7 × 83	**b. 8 × 56**	**c. 5 × 216**
7 × ____ + 7 × ___	___ × ____ + ___ × ____	5 × ____ + 5 × ____ + 5 × ___
____ + ____	____ + ____	____ + ____ + ____
= ______	= ______	= ______

2. Multiply using partial products.

a.

$$\begin{array}{rr} & 4\ 9\ 2 \\ & \times \quad 6 \\ \hline 6 \times 2 \rightarrow & \\ 6 \times 90 \rightarrow & \\ 6 \times 400 \rightarrow & + \\ \hline & \end{array}$$

b.

$$\begin{array}{r} 2\ 5\ 5 \\ \times \quad 4 \\ \hline \\ + \\ \hline \end{array}$$

c.

$$\begin{array}{r} 8\ 1\ 7 \\ \times \quad 7 \\ \hline \\ + \\ \hline \end{array}$$

3. Multiply bigger numbers using partial products.

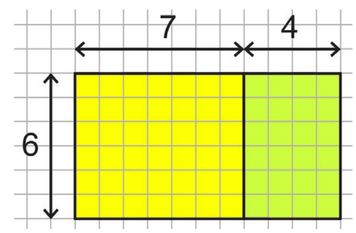
a.	b.	c.
$\begin{array}{r} 2\;5\;1\;0 \\ \times \quad\;\; 9 \\ \hline \end{array}$	$\begin{array}{r} 4\;4\;7\;8 \\ \times \quad\;\; 5 \\ \hline \end{array}$	$\begin{array}{r} 2\;6\;0\;7\;2 \\ \times \qquad\; 6 \\ \hline \end{array}$
$9 \times 0 \rightarrow$		
$9 \times 10 \rightarrow$		
$9 \times 500 \rightarrow$		
$9 \times 2{,}000 \rightarrow$ + ______	+ ______	+ ______

Remember? The picture on the right illustrates a principle that ties together area, addition, and multiplication.

The total area is 6 × (7 + 4) square units.

The areas of the two parts are 6 × 7 and 6 × 4.

Therefore, 6 × (7 + 4) equals 6 × 7 + 6 × 4.



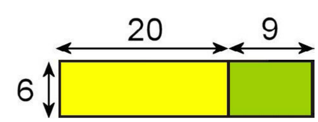
6 × (7 + 4) = 6 × 7 + 6 × 4

This principle is called *the distributive property*, because it "distributes" multiplication over addition. In general, we can express it using symbols: $a \times (b + c) = a \times b + a \times c$.

4. Fill in the missing parts, thinking of the area of the whole rectangle, or of the partial rectangles.

a. 6 × 29 = 6 × _____ + 6 × _______

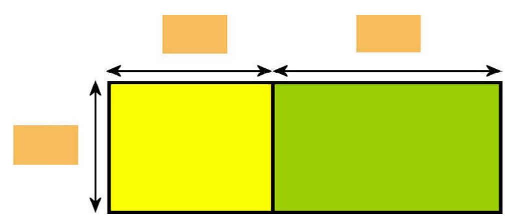
= ________ + __________ = _________



b. 8 × (10 + 14)

= 8 × _____ + 8 × ______

= _________ + _________ = _________

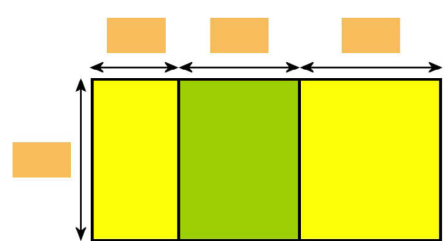


c. 11 × (3 + 5 + 6)

= _____ × ____ + _____ × ____ + _____ × ____

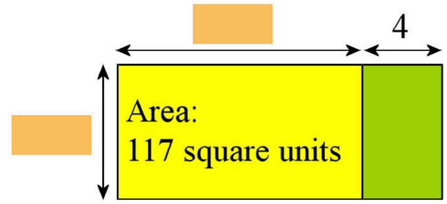
= __________ + __________ + __________

= _________



5. The total area of this figure is 153 square units, and the area of the yellow part is 117 square units.

a. What *other* area can you find out using the two given areas (153 and 117)?



b. Find the missing lengths of the sides.

6. Use partial products and mental math to solve the problems:

a. What is the total cost if you buy seven hammers costing $26 each?

b. Paul is a truck driver. One work day, he ended up making three round trips between two towns that are 113 km apart. What was the total distance he drove?

7. Which expression or expressions match the problem? *You do not have to calculate the answer.*

Paul bought 26 algebra textbooks for $18 each and 26 workbooks for $8 each. What was the total cost?

a. 26 × $18 + $8	**b.** 26 × $26
c. 26 × $18 + 26 × $8	**d.** 26 × ($18 + $8)

8. For each two expressions, decide if the answers are the same or not. Do *not* calculate the answers.

a. 5 × 37 + 4 × 37 6 × 37	**b.** 9 × 28 + 7 × 28 6 × 28 + 10 × 28	**c.** 6 × 128 6 × 120 + 8
d. 57 × 89 + 3 × 89 60 × 89	**e.** 8 × 76 – 5 × 76 2 × 76	**f.** 33 × 45 – 45 32 × 45

The Multiplication Algorithm

An *algorithm* is a step-by-step method for solving a particular kind of problem.

In this lesson we practice **the standard multiplication algorithm**, which you already know from 4th grade.

This algorithm is based on multiplying in parts. For example, 7 × 648 is done in three parts: 7 × 600, 7 × 40, and 7 × 8. At each step, you may need to regroup and add.

$$\begin{array}{r} {}^{5} \\ 6\;4\;\mathbf{8} \\ \times\;\;\;\mathbf{7} \\ \hline \mathbf{6} \end{array} \qquad \begin{array}{r} {}^{3}\;{}^{5}\;\; \\ 6\;\mathbf{4}\;8 \\ \times\;\;\;\mathbf{7} \\ \hline \mathbf{3}\;6 \end{array} \qquad \begin{array}{r} {}^{3}\;{}^{5}\;\; \\ \mathbf{6}\;4\;8 \\ \times\;\;\;\mathbf{7} \\ \hline \mathbf{4}\;\mathbf{5}\;3\;6 \end{array}$$

$7 \times 8 = 56$ $\qquad$ $7 \times 4 + 5 = 33$ $\qquad$ $7 \times 6 + 3 = 45$

1. Review your multiplication skills.

a. $\begin{array}{r} 4\;1\;5 \\ \times\;\;\;8 \\ \hline \end{array}$ **b.** $\begin{array}{r} 8\;7\;7 \\ \times\;\;\;8 \\ \hline \end{array}$ **c.** $\begin{array}{r} 1\;7\;5\;2 \\ \times\;\;\;7 \\ \hline \end{array}$ **d.** $\begin{array}{r} 2\;6\;1\;5 \\ \times\;\;\;4 \\ \hline \end{array}$

The process is the same with more digits. Study the example.

$$\begin{array}{r} {}^{4}\;\; \\ 6\;1\;3\;5\;\mathbf{9} \\ \times\;\;\;\mathbf{5} \\ \hline \mathbf{5} \end{array} \quad \begin{array}{r} {}^{2}\;{}^{4}\;\; \\ 6\;1\;3\;\mathbf{5}\;9 \\ \times\;\;\;\mathbf{5} \\ \hline \mathbf{9}\;5 \end{array} \quad \begin{array}{r} {}^{1}\;{}^{2}\;{}^{4}\;\; \\ 6\;1\;\mathbf{3}\;5\;9 \\ \times\;\;\;\mathbf{5} \\ \hline \mathbf{7}\;9\;5 \end{array} \quad \begin{array}{r} {}^{1}\;{}^{2}\;{}^{4}\;\; \\ 6\;\mathbf{1}\;3\;5\;9 \\ \times\;\;\;\mathbf{5} \\ \hline \mathbf{6}\;7\;9\;5 \end{array} \quad \begin{array}{r} {}^{1}\;{}^{2}\;{}^{4}\;\; \\ \mathbf{6}\;1\;3\;5\;9 \\ \times\;\;\;\mathbf{5} \\ \hline \mathbf{3}\;\mathbf{0}\;6\;7\;9\;5 \end{array}$$

$5 \times 9 = 45$ $\qquad$ $5 \times 5 + 4 = 29$ $\qquad$ $5 \times 3 + 2 = 17$ $\qquad$ $5 \times 1 + 1 = 6$ $\qquad$ $5 \times 6 = 30$

2. Multiply 5- and 6-digit numbers.

a. $\begin{array}{r} 1\;7\;5\;5\;2 \\ \times\;\;\;7 \\ \hline \end{array}$	**b.** $\begin{array}{r} 2\;7\;8\;0\;5 \\ \times\;\;\;3 \\ \hline \end{array}$	**c.** $\begin{array}{r} 1\;4\;4\;1\;2\;3 \\ \times\;\;\;5 \\ \hline \end{array}$
d. $\begin{array}{r} 2\;7\;0\;8\;1\;4 \\ \times\;\;\;3 \\ \hline \end{array}$	**e.** $\begin{array}{r} 5\;1\;6\;2\;0 \\ \times\;\;\;9 \\ \hline \end{array}$	**f.** $\begin{array}{r} 2\;3\;9\;3\;1\;3 \\ \times\;\;\;4 \\ \hline \end{array}$

Estimate before you multiply. Then compare your estimated result with the final result, and that way you may catch some gross errors.

3 × 21,578 = ?

Round 21,578 in such a way that you can easily multiply in your head. It makes sense to round it to 22,000.

Estimate: 3 × 22,000 = 66,000

The exact result is 64,734. The estimate is quite close.

$$\begin{array}{r} {}^{1\,2\,2} \\ 2\;1\;5\;7\;8 \\ \times \qquad 3 \\ \hline 6\;4\;7\;3\;4 \end{array}$$

3. First estimate, by rounding the number in such a way that you can multiply in your head. Then multiply. Check that your final answer is reasonably close to your estimate.

a. Estimate: 5 × 8,871 ≈ ____________

Calculate exactly:

$$\begin{array}{r} 8\;8\;7\;1 \\ \times \qquad 5 \\ \hline \end{array}$$

b. Estimate: 4 × 22,399 ≈ ______________

Calculate exactly:

$$\begin{array}{r} 2\;2\;3\;9\;9 \\ \times \qquad 4 \\ \hline \end{array}$$

c. Estimate: 7 × 87,240

≈ ____________________________________

Calculate exactly:

d. Estimate: 4 × 212,788

≈ ____________________________________

Calculate exactly:

4. Jenny's estimate for the problem 3 × 173,039 is quite far from her final answer. Figure out where Jenny makes an error or errors.

Jenny's estimate:

3 × 173,039
≈ 3 × 170,000
= 510,000

Jenny's calculation:

$$\begin{array}{r} {}^{1\;2\;\;\;\;} \\ 1\;7\;3\;0\;3\;9 \\ \times \qquad\quad 3 \\ \hline 4\;2\;9\;0\;1\;7 \end{array}$$

A Two-Digit Divisor

Long division works exactly the same way with two-digit divisors as with single-digit divisors. However, since a lot of us cannot quickly multiply mentally by two-digit numbers, it is often helpful to write the multiplication table of the divisor before you divide.

Example 1. This division is by 30, which makes it easy, because the multiplications will be easy to do in one's mind.	0 2 30) 7 2 6 4 30 goes into 7 zero times, so we look at 72. How many times does 30 go into 72? Two times, because 2 × 30 = 60, and 3 × 30 = 90.	0 2 4 30) 7 2 6 4 -6 0 1 2 6 Now, how many times does 30 go into 126? Since 4 × 30 = 120, it is four times.	0 2 4 2 30) 7 2 6 4 -6 0 1 2 6 -1 2 0 6 4 - 6 0 4 Lastly, 30 goes into 64 two times, and there is a remainder of 4.
Example 2. This division is by 16, so we will write the multiplication table of 16: 3 × 16 = 48 4 × 16 = 64 5 × 16 = 80 6 × 16 = 96 7 × 16 = 112 8 × 16 = 128 9 × 16 = 144	0 3 16) 5 5 6 8 16 goes into 5 zero times, so we look at 55. How many times does 16 go into 55? Check in the table on the left. We see it goes into 55 three times.	0 3 4 16) 5 5 6 8 -4 8 7 6 Now, how many times does 16 go into 76? From the table we can see that it is four times.	0 3 4 8 16) 5 5 6 8 -4 8 7 6 - 6 4 1 2 8 - 1 2 8 0 Lastly, 16 goes into 128 exactly 8 times, and the division is over.

1. Divide. Check each answer by multiplying.

a. 40) 8 6 4 0

b. 50) 6 1 9 3

2. Divide. Check each answer by multiplying.

a.

$$70 \overline{)9852}$$

b.

$$12 \overline{)888}$$

3. Divide. Writing a list of multiples of the divisor can help. Check each answer by multiplying.

a.

$$21 \overline{)3822}$$

2 × 21 =
3 × 21 =
4 × 21 =
5 × 21 =
6 × 21 =
7 × 21 =
8 × 21 =
9 × 21 =

b.

$$15 \overline{)4815}$$

2 × 15 =
3 × 15 =
4 × 15 =
5 × 15 =
6 × 15 =
7 × 15 =
8 × 15 =
9 × 15 =

c.

$$25 \overline{)6275}$$

2 × 25 =
3 × 25 =
4 × 25 =
5 × 25 =
6 × 25 =
7 × 25 =
8 × 25 =
9 × 25 =

d.

$$16 \overline{)1504}$$

2 × 16 =
3 × 16 =
4 × 16 =
5 × 16 =
6 × 16 =
7 × 16 =
8 × 16 =
9 × 16 =