High School Students

Working with Rates of Change

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

Based on projections from 2006–07 to 2016–17, the number of students in grades 9 through 12 in the United States may be modeled by

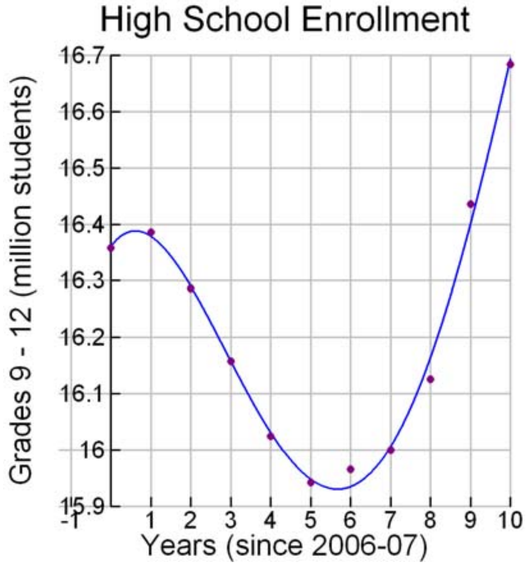
$$s(t) = -0.4936t^4 + 13.27t^3 - 89.50t^2 + 95.10t + 16,360 \text{ thousand students}$$

where t is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

2. Use technology and the derivative of s to find the absolute extrema of s on the interval $[0,10]$. Then interpret the real world meaning of each extreme value.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

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1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

$$s'(t) = -1.9744t^3 + 39.81t^2 - 179.0t + 95.10$$

$$\begin{aligned} s'(3) &= -1.9744(3)^3 + 39.81(3)^2 - 179.0(3) + 95.10 \\ &\approx -136.9 \textit{ thousand students per year} \end{aligned}$$

$$\begin{aligned} s'(8) &= -1.9744(8)^3 + 39.81(8)^2 - 179.0(8) + 95.10 \\ &\approx 200.0 \textit{ thousand students per year} \end{aligned}$$

According to the model, the number of students will be decreasing at a rate of 136.9 thousand students per year in 2009 – 10 and increasing at a rate of 200.0 thousand students per year in 2014 – 15. The number of students is changing more rapidly in 2014 – 15.

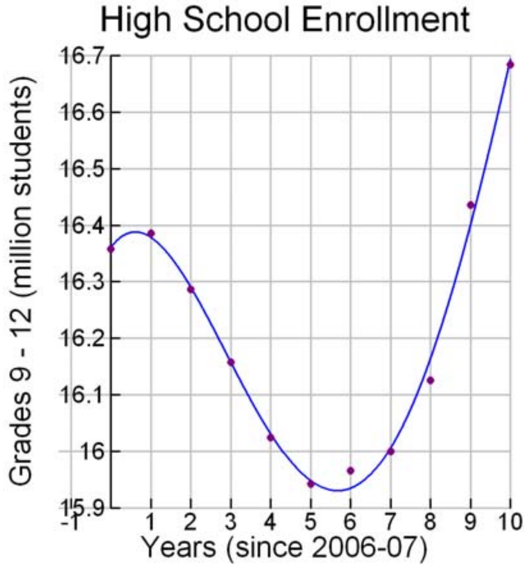
2. Use technology and the derivative of s to find the absolute extrema of s on the interval $[0,10]$. Then interpret the real world meaning of each extreme value.

Relative extrema of s *occur wherever* s' *changes sign. We graph* $s'(t)$ *and* $y = 0$ *simultaneously and find the points of intersection to determine the critical values of* s*. The critical values of* s *are* $s \approx 0.61$ *and* $s = 5.67$.

t	*s*
0	*16,360*
0.61	*16,390*
5.67	*15,930*
10	*16,700*

Rounded to the nearest multiple of 10 thousand, the absolute maximum number of students enrolled will be 16,700 thousand and the absolute minimum number of students enrolled will be 15,930 thousand.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



We are looking for where the graph changes concavity. It appears that the graph changes from concave down to concave up at around $(3,16.15)$. So the inflection point of s is approximately $(3,16150)$. That is, in 2009 – 2010 the number of students is about 16,150,000. In terms of the rate of change of s, this is the year when the number of students is decreasing most rapidly.

4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

$$s(0) = -0.4936(0)^4 + 13.27(0)^3 - 89.50(0)^2 + 95.10(0) + 16,360$$
$$= 16,360$$

$$s(10) = -0.4936(10)^4 + 13.27(10)^3 - 89.50(10)^2 + 95.10(10) + 16,360$$
$$= 16,695$$

$$\frac{s(10)-s(0)}{10-0} = \frac{16,695-16,360}{10-0}\frac{\textit{thousand students}}{\textit{year}}$$
$$= 33.5\,\textit{thousand students per year}$$

Between 2006 – 07 and 2016 – 17, the number of students in grades 9 – 12 was expected to increase by an average of 33,500 students per year. That is, although the number of students increased and decreased over the ten year period, the average of the rates of change was 33,500 students per year.

Worksheet Title	High School Students: Working with Rates of Change			*Filename:*	m1008
Keywords	9 – 12, grades, high school, students, enrollment, derivative, rate of change, calculus, inflection point, concavity				
NCTM Standard		Content Standards			Process Standards
		Number and Operations		X	Problem Solving
	X	Algebra		X	Reasoning and Proof
		Geometry		X	Communication
		Measurement		X	Connections
		Data Analysis and Probability			Representations
Grade Band		PreK – 2			
		3 – 5			
		6 – 8			
	X	9 – 12			
	X	College			
Data Type	Equation, graph				

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